

NASA TT F-9772

|                   |                               |            |
|-------------------|-------------------------------|------------|
| FACILITY FORM 602 | ELECTROCARDIOGRAPHY           |            |
|                   | N66 24908                     |            |
|                   | (ACCESSION NUMBER)            | (THRU)     |
|                   | 195                           | 1          |
|                   | (PAGES)                       | (CODE)     |
|                   |                               | 04         |
|                   | (NASA CR OR TMX OR AD NUMBER) | (CATEGORY) |

H. C. Burger

Editor: H. W. Julius, Jr.

Translation of "De elektrische werking van het hart"  
1963

GPO PRICE \$ \_\_\_\_\_

CFSTI PRICE(S) \$ \_\_\_\_\_

Hard copy (HC) 5.00Microfiche (MF) 1.25

# 653 July 65

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
WASHINGTON  
DECEMBER 1965

## CONTENTS

| Chapter                                                            | Page |
|--------------------------------------------------------------------|------|
| 1. Introduction                                                    | 1    |
| 2. Anatomy and Histology of the Heart                              | 3    |
| Anatomy                                                            | 3    |
| Histology                                                          | 7    |
| 3. Instrumentation                                                 | 13   |
| Introduction                                                       | 13   |
| Historical Review                                                  | 17   |
| Recording Technique                                                | 20   |
| 4. Placing the Electrodes and EKG                                  | 23   |
| 5. The Electric Dipole                                             | 29   |
| 6. Einthoven's Triangle                                            | 39   |
| 7. Connection Between Heart Vector and Leads. The Vectorcardiogram | 50   |
| 8. Geometrical Representation                                      | 66   |
| 9. Application of the Geometric Representation                     | 78   |
| 10. Testing the Dipole Approximation                               | 82   |
| Cancellation Method                                                | 82   |
| Becking's Method of Cancellation                                   | 86   |
| Transformation Method                                              | 90   |

| Chapter                                                               | Page |
|-----------------------------------------------------------------------|------|
| 11. The Total Dipole and Corrected Systems                            | 95   |
| The BW System (Burger)                                                | 104  |
| The F System (Frank)                                                  | 108  |
| The S System (Schmitt)                                                | 109  |
| The M System (McFee)                                                  | 109  |
| 12. Subjective and Objective Comparison of Vectorcardiography Systems |      |
| Compromise in Vectorcardiography                                      | 119  |
| Subjective Comparison                                                 | 119  |
| Objective Comparison                                                  | 122  |
| 13. The Pole Vector ( $\vec{P}$ )                                     | 134  |
| 14. The Ventricular Gradient ( $\vec{G}$ )                            | 138  |
| 15. The Multipole Effect                                              | 145  |
| 16. The Origin of the Electric Activity of the Heart                  | 158  |
| 17. Clinical Application of Vectorcardiography                        | 167  |
| General                                                               | 167  |
| The Normal VKG                                                        | 172  |
| The Pathological VKG                                                  | 173  |

## CHAPTER 1.

## Introduction

Years ago the discovery of the electrical phenomena associated with /1\* the heart beat led to electrocardiography (EKG): a physical method at the disposal of the clinic which enables us to make a judgment concerning heart function, particularly pathological aberrations thereof.

The point of departure is the measurement of differences of potential between various locations on the surface of the human body as a function of time: the electrocardiogram (EKG).

In addition to "normal" electrocardiography, vectorcardiography (VKG), which is a variant on the "classic concept", will be considered in more detail. It is not a special entity but rather is woven into the whole fabric, in consideration of the fact that there is no distinctly discernable boundary between the two methods.

There are two different ways of attacking a problem: the analytic and the synthetic. We shall choose the first and begin with phenomena, and from thence penetrate more deeply to the core of the matter: the heart as source of electric phenomena.

The treatment of a subject like electrocardiography, which medical men as well as physicists have worked on, and over which in the course of years

---

\*Numbers given in the margin indicate the pagination in the original foreign text.

many differences of opinion and misconceptions have arisen and still exist (as a result of the fact that biological and exact physical concepts, especially on the part of the nonphysicist, are so difficult to combine) makes it impossible to escape historical considerations. This not only affords the occasion for following the development at many points, but also for critical examination of statements in the literature.

Before we discuss the physical problems relating to electrocardiography and vectorcardiography, we shall consider the topic of our study--the heart--from the point of view of anatomy.

## CHAPTER 2

### Anatomy and Histology of the Heart

#### I. Anatomy

In the heart, which <sup>hardly</sup> can be described as heart-shaped, there are 2 distinguished right and left sections--very unequal in size, since the left

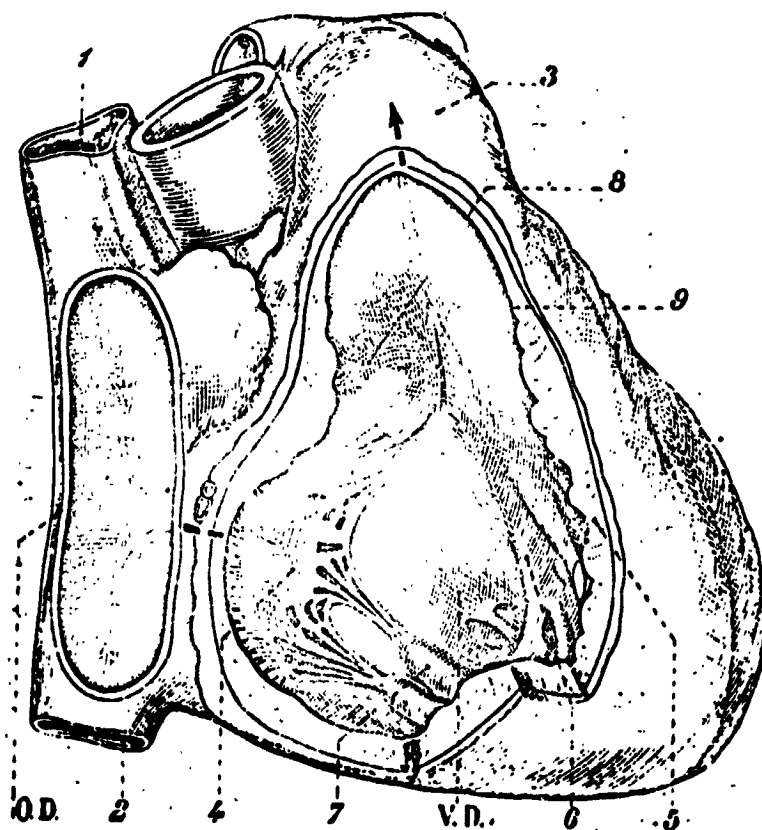


Figure 1. Heart as seen from the right, part of the wall of ~~the~~ right atrium (O.D.) and ventricle (V.D.) are cut away.

1 and 2, vena cava superior and inferior; 3, aorta pulmonalis;  
4, tricuspid valve; 7, papillary muscle.

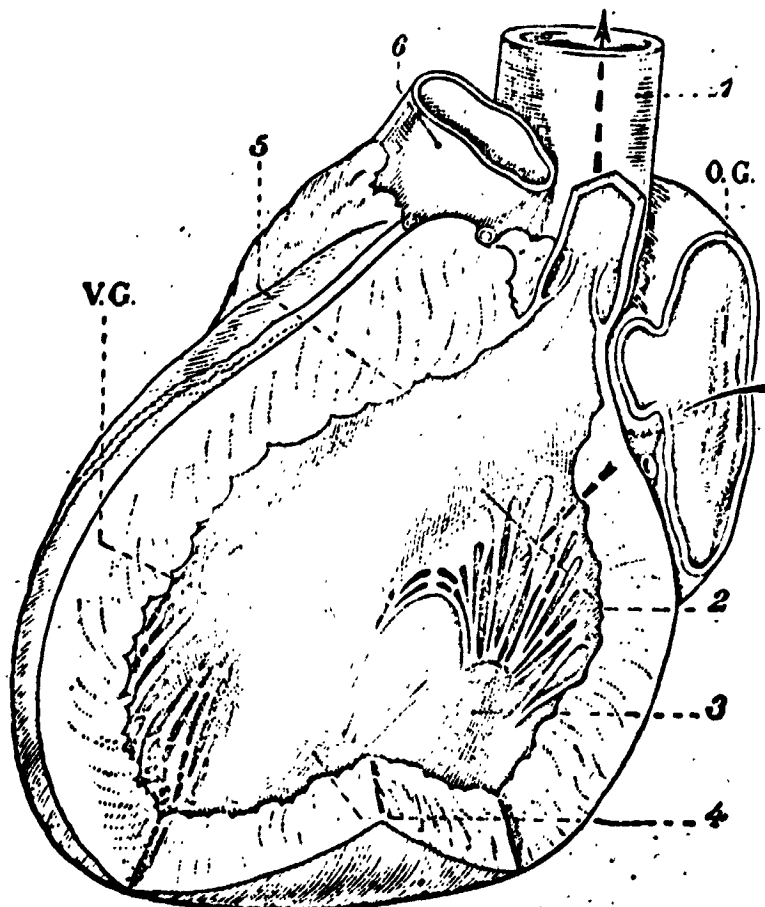


Figure 2. Heart seen from the left, part of left atrium (O.G.) and ventricle (V.C.) removed. 1, aorta with aortic valve; 2, bicuspid valve.

portion is more sturdily built--and in each part there is distinguished an atrium and a ventricle. The left and right atria and ventricles are separated from each other by the septum auricularum and the septum ventriculorum, respectively. Blood flows into the right atrium from the venae cavae from the body circulation. The blood has access to the chamber via the membranous connective tissue valve, the tricuspid, during the diastole of the right atrium when the muscle wall is flaccid. The flow from atrium to ventricle is facilitated by a contraction of the wall of the atrium, but it cannot be described as a pumping action because there is no valve to check a backward flow of blood

to the blood vessels. When the diastole of the ventricle is completed, the tricuspid valve closes by contraction (systole). It is protected against being blown out because it is attached to the atrium wall by tendinous strings (chordae tendineae). These strings are fixed at the top of the conical muscles (papillary muscles) which are attached at the top of the atrium.

By their contraction simultaneously with that of the rest of the musculature of the atrium wall, these papillary muscles compensate for the approach of the chamber wall to the level of the valve. A rise of the blood pressure in the ventricle above that in the arteria pulmonalis opens the semilunar valve at the origin of this artery and the chamber is emptied. Via the pulmonary artery, the lungs and the venae pulmonales the blood reaches the left atrium. There takes place here an alternation of diastole and systole, instead of contraction simultaneously with that on the right, so that the blood is finally expelled from the aorta. Differing from those on the right, the valves between the left atrium and ventricle have two flaps (valvulae bicuspidales). Moreover the left ventricle in conformity with the greater resistance in the body circulation is much thicker-walled than the right. The latter, in transverse section through the ventricular muscle, does not look like much more than a recess in the wall of the former.

The ventricular muscle which is far and away the major part of the heart, has the form of an inverted pear. The truncated end is a circular plate of connective tissue (the annulus fibrosus) wherein the openings of the tricuspid, bicuspid and semilunar valves are clustered together.

The wall of the heart is lined on the inside by a smooth membrane, the endocardium, and on the outside it is covered by a similar membrane, the

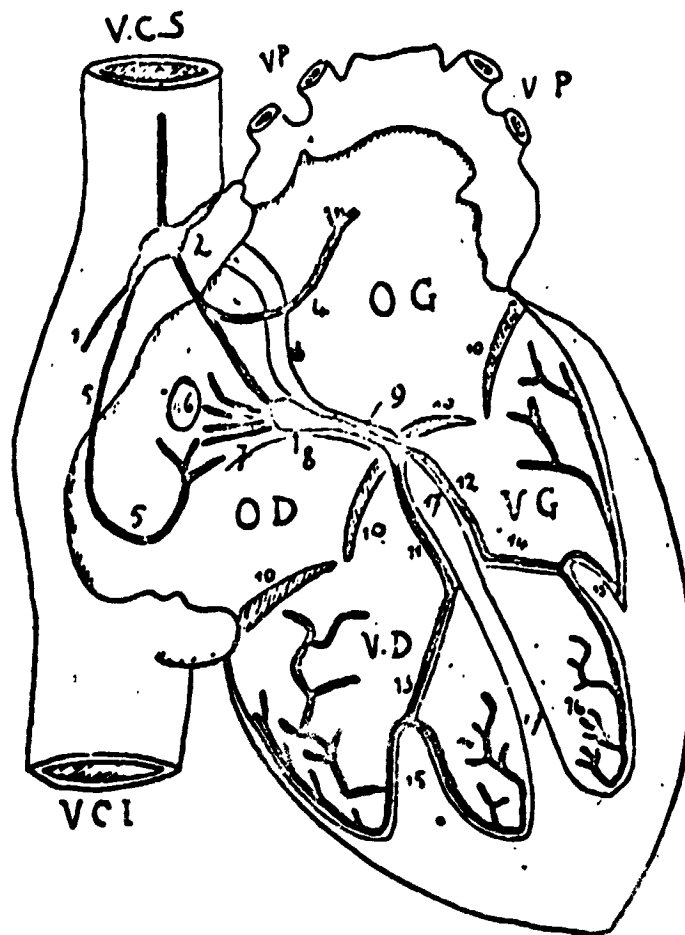


Figure 3. Schema of the impulse-transmitting system.

- 2, Sinoauricular node (Keith-Flack), Tawara's node;  
 9, common part of the bundle of His-Tawara, 11 and  
 12, right and left branches.

epicardium. The epicardium is separated from the smooth pericardium (heart sac) by a thin membranous layer. Through this construction, the ventricular part of the heart has great freedom of motion.

The myocardium is supplied with blood through the arteriae coronariae which start at the point of origin of the aorta, and pass down just under the epicardium and extend branches into the depths of the muscular tissue.

On the inside of the right atrium and on either side of the septum between the ventricles it is possible to discern, from the direction of the endocardium, a system of light-colored fibers which at first are joined together as bundles but which spread out in the ventricle and disappear in the myocardial tissue. On the basis of physiological observation they are designated the impulse-transmitting system. The system includes the sinoauricular node (Keith-Flack), the node of Tawara and the His-Tawara bundle which divides into a right and left descending branch along the inside of the septum ventriculorum.

## II. Histology

In the anatomical description, reference was made to muscular and connective tissue. There is a great difference between these two types of tissue. Connective tissue is formed, it is true (and also maintained) by the metabolizing cells, but the mechanical properties are due entirely to the intercellular constituents: collagen, elastic fibers and interfibrillar substance. These three components exist in varying combinations and proportions. Collagen fibrils have great tensile strength, and have little elastic protein filament. They determine the solidity of tendons, heart valves, chordae tendineae and annulus fibrosus. /5

In these tissues 90 percent of the volume is occupied by the collagen fibrils, the rest by interfibrillar material (preponderantly a carbohydrate derivative) and the individual connective tissue cells (fibroblasts). Elastic fibers are not nearly as strong as collagen, but they can be stretched by 100 to 200 percent. They constitute the main body of the wall of the great arteries which function as air chambers. Muscular tissue derives embryologically from /6

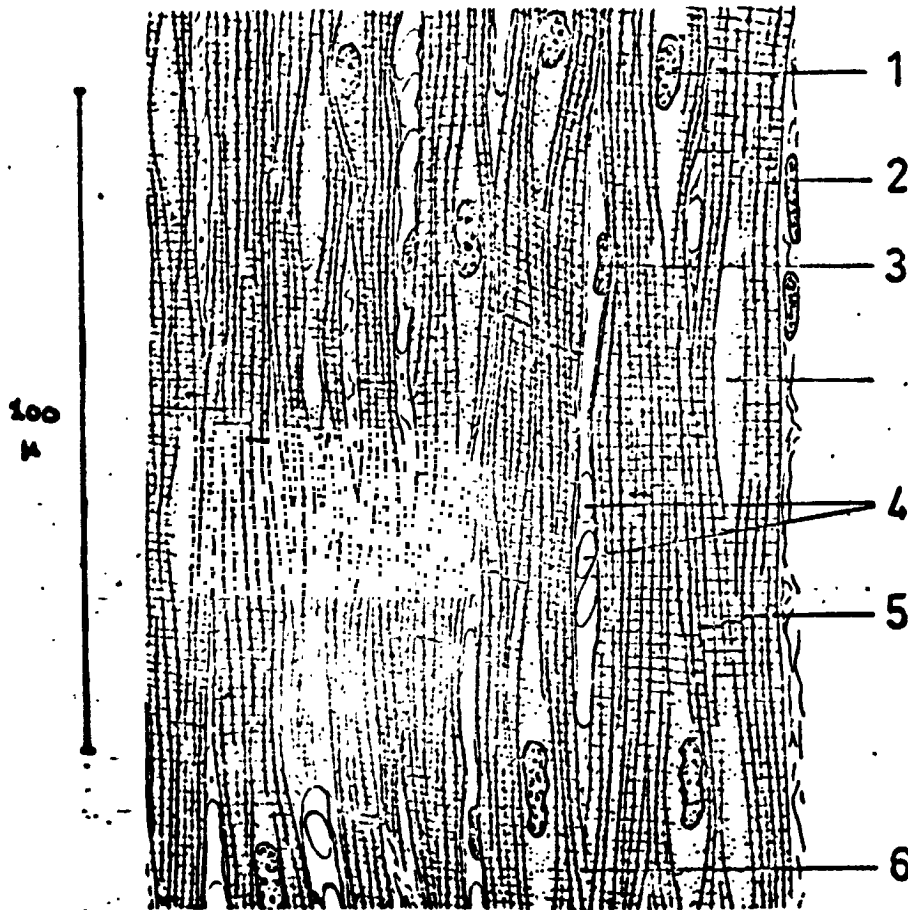


Figure 4. Sketch of a cross section of the myocardium, parallel to the long axis of the muscle fibers. 1, nucleus of a muscle cell; 2 and 3, nuclei of connective tissue and capillary cells; 4, erythrocytes in the capillary; 5, cell boundary; 6, branching of a muscle cell.

the same embryonic tissue as connective tissue, but whereas in connective tissue the fibers between the cells determine the character, muscular tissue is essentially cellular. The muscle fibers are nothing other than the cells of the tissue in which the fine collagen fibrils that interconnect the muscle cells (the endomysium) are reduced to a very minor part of the volume. <sup>21</sup>

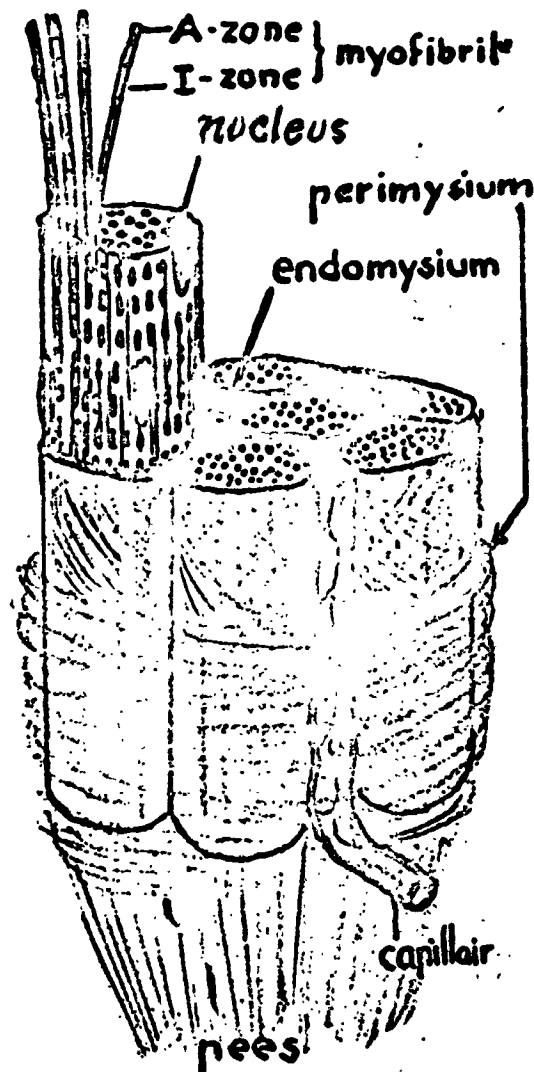


Figure 5. Schema of the end of a single skeletal muscle fiber (Dr. van den Broeck, Laboratory of Histology).

There are two kinds of muscular tissues: striated (skeletal musculature ("meat"), myocardium) and smooth (intestinal musculature, musculature of the more peripherally located arteries).

Myocardial tissue is composed of remarkably formed cells which by interconnection form a dense, taut three-dimensional network. (The interstices are filled with capillaries which are supplied by the coronary arteries.) In skeletal muscle, the muscle fibers are long protoplasm structures with many cell

nuclei. A skeletal muscle fiber is thus not the equivalent of a single cell with a single nucleus like myocardial fibers.

The following features are found in both types of muscle (see fig. 5).

The muscle fiber has a sheath, the sarcolemma, which consists of a /8 layer of fine collagen fibrils embedded in interfibrillar substance and in a special cell membrane. This encloses the cytoplasm which in this case is designated sarcoplasm. In the sarcoplasm (thus not, as in the connective tissue in the intercellular space) the myofibrils run in the direction of the long axis of the fiber. These myofibrils are the true contractile elements. They are segmented at specific distances, with a repeated difference in optical properties (isotropic: I-zone, anisotropic: A-zone), staining properties etc. Moreover, corresponding segments of the myofibrils lie side by side in a single fiber so that the segmentation of the myofibrils gives the whole muscle fiber a striped appearance. This effect is accentuated by the presence of a transverse partition in the muscle fiber, the Z-bands (Krause's membranes: Tr.). The myofibrils penetrate these membranes at the height of the middle of their isotropic segments. The multiply repeated unit of a muscle fiber consisting of Z-band, half I-zone, A-zone-half I-zone, Z-band, is considered to be a sarcomere.

The myofibrils are optically distinguishable. In electron micro- /9scopic examination it appears that each myofibril is a bundle of much finer fibrils, called myofilaments. There are two recognizable types: the thick ones which are restricted to the A-zone, and thin ones which pass through the I-zone on each side of the Z-band and which finally pass between the thick filaments in the A-zone. In contraction the thin filaments apparently are thrust in between the thick A-zone filaments.

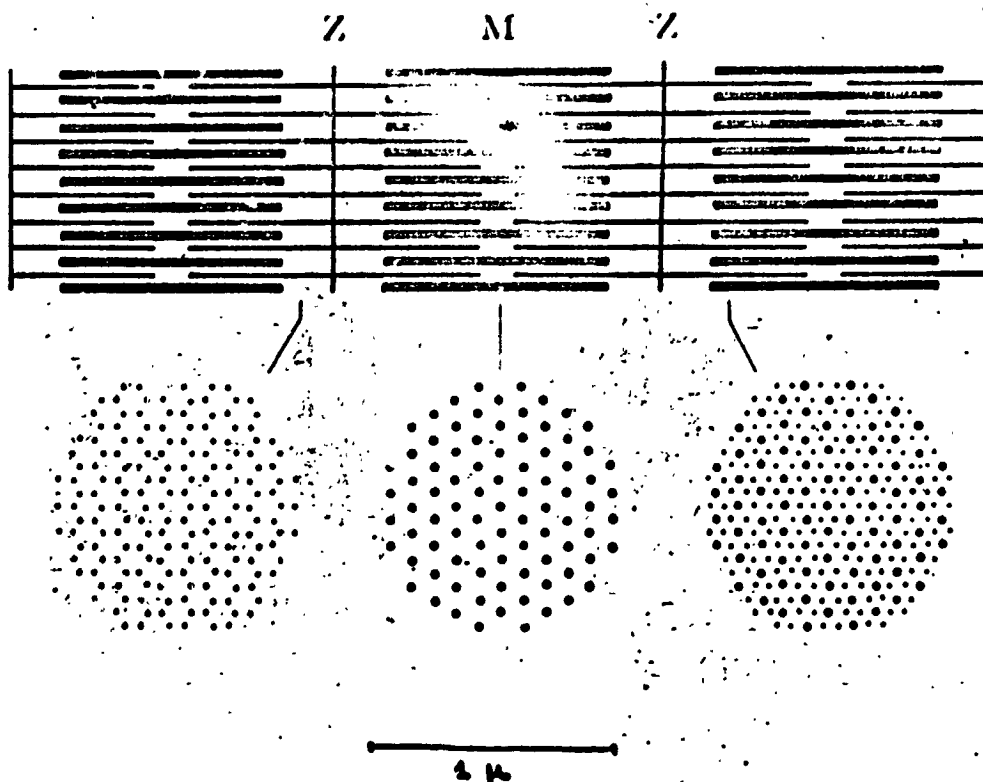


Figure 6. Schema of myofilaments in a myofibril, above in longitudinal section, below in transverse section (electron microscope photos). M, "middle disk," the center part of the A-zone. Z = Z-band.

The impulse for contraction of the myofibrils is given by the transmitted depolarization of the cell membrane. This electric phenomenon which is basic to electrocardiography will be discussed elsewhere. Electron microscopy has produced evidence that the cell membrane at the place where the Z-bands are attached presents depressions and that the entire Z-band constitutes a system of tubes that extends deeply into the sarcoplasm between the myofibrils. It has been hypothesized that the membrane of the canal system in the interior of the fiber plays the role that--at the surface of the fiber--is assigned to the cell membrane, i.e., transmission of the stimulus. Only in this way would it be possible for the peripheral and central myofibrils of a muscle fiber to respond synchronously to the stimulus.

The fibers of the impulse-transmitting system and the nodes of Keith-Flack and Tawara are in principle myocardial fibers, and hence they have a great deal of sarcoplasm and myofibrils only irregularly. The transmission of impulses occurs along these fibers approximately ten times as rapidly as in normal myocardial fibers.

Neural fibers are provided for the whole myocardium and especially for the impulse-transmitting system, but it is not believed that normal transmission of impulses in the myocardium takes place via the neural fibers.

## CHAPTER 3

### Instrumentation

#### A. Introduction

If we assume that as a result of the emanation of electricity from /10 the heart, an electrical phenomenon occurs at the body surface which varies in place and time during each heart beat, then we must first answer the question: must we measure current or voltage? The years-long argument on this point is decided in favor of the latter. We can only measure voltage! For measurement of a current, the circuit must be interrupted to introduce a measuring instrument, and this is not possible with a medium such as the human body.

Consequently we measure voltages, hence differences in potential. In this case also history sets us an example with a misconception: next to the question concerning the difference of potentials between the right arm (R) and the left arm (L) there was the question of the individual potential. It must be obvious to us all that the answer brings us no farther along. Because, if a test individual could be exposed to a million grounded volts, there would be no effect on the difference of potential between R and L, and thus the amount of information obtained from the potential itself would be no greater!

Yet those who sought "a potential" had such great influence that even today various methods are used whereby potential differences are measured between all sorts of places on the body and one meticulously selected reference point at which one constant or even one potential is valued at zero (see page /19).<sup>1</sup>

<sup>1</sup>Page references in text refer to original language pages indicated in right-hand margin.

2

Before we go more deeply into the methodological and theoretical background, we will first consider the apparatus which is used.

An electrocardiograph is essentially a voltmeter which must meet a number of requirements. We shall discuss these briefly.

#### 1. Satisfactory sensitivity

The voltages <sup>with</sup> which we are to be concerned are maximally on the order /11 of 1 millivolt. The sensitivity of the voltmeter, defined as the deflection (in mm) of the indicator per unit of voltage (expressed in millivolts) must be adjusted to these values. In order to make the comparison between EKGs as easy as possible, a uniform scale must be selected: 1 millivolt  $\sim$  10 mm (see page 30). As a matter of fact, this scale is now too small: it appears that certain details are lost.

#### 2. High input impedance

This requirement essentially is inherent for all voltmeters. In the electrocardiograph this requirement is basic. If the total resistance is too small, a significant current can go astray, which at the measuring point yields a voltage drop dependent upon the changing resistance of the skin, and this latter is very high (on the order of  $10^5 \Omega \cdot \text{cm}^2$ ). Various methods have been tried to reduce the skin resistivity: skin-active pastes, abrasion of the skin etc. The skin electronic apparatus nevertheless permits such high input impedances ( $\pm 10 \text{ M}\Omega$ ) that it seems to be sufficient to saturate the electrodes (furnished with small foam rubber pads) with NaCl solution which penetrates the outer layer of the skin and thus effects an electrolytic contact.

In addition to the ohmic resistance obtained, another phenomenon makes a high input resistance necessary, and that is electrolytic polarization. This causes a counteracting electromotive force on the surface of the electrodes which in good approximation is proportional to the total amount of electricity flowing through.

### 3. Short indicating time

Although a complete heart cycle takes about one second, the different changes in this period occur so rapidly that the requirement for rapidity in the instrument is high. This property is needed not only for the indicating apparatus but also for the electronic part of it (amplifier), so that fast voltage changes can be transmitted without distortion. It is possible to ask at what frequency an undistorted reproduction can be achieved.

Opinions on this are divergent, but it is industry that "helps" us /12 out of the impasse here and in practice seldom sets the limit above 500 to 1000 cycles. If anybody can obtain any information up there...?

The frequency and jump characteristics offer complete information on the performance of the apparatus in this respect.

Of course, a fast instrument alone is not enough: the speed of film or paper strip must be so adjusted that quick changes are actually recorded. Here also a uniform and generally satisfactory standard is required: 25 mm/sec (see page 30). In special instances this rate will be increased by a factor of 2, 4 or even 10.

Finally it must be noted that standards must also be set for line sharpness in general, and with pen or jet tracing for line thickness. The thinner the line, the more data in proportion to the area of the paper.

#### 4. Freedom from disturbance (Artifacts)

The requirement, freedom from disturbance, is naturally set for all instruments of physical measurement. Instead of going deeply into the general topic, we will indicate the individual kinds of disturbance that are significant in EKG apparatus.

##### a. Electric disturbances

In principle these can be overcome in large measure today by the electronic part of the apparatus by the balanced input. Practice proves however that something is still missing. A solution, but in the clinic what is no less difficult a problem is the surrounding of the patient with the electrodes, leads and preamplifier by a Faraday cage.

##### b. Magnetic disturbances

Caused for example by rotating motors. Here also as under section(a) a screen must be constructed of a material with high magnetic permeability, though the high cost makes this practically impossible. Besides, the patient through his anxiety reaction will be able to introduce more disturbance than we are attempting to eliminate! (See(d)). The measures that can be taken are: remain as far as possible from the source, avoid use of straps, and in application of "counter-straps" try for a compensation. Sometimes only one measure helps: cut off the source.

/13

##### c. Building vibrations

These can be transmitted to the recorder by the tubes of the electronic apparatus.

##### d. Muscular movement

Not only heart activity gives rise to electric voltages: the activity of all muscles is associated with an electric phenomenon. If only the first signals

are to be recorded, care must be taken to see that the patient is as relaxed as possible. He should be allowed to lie down and be placed as comfortably as possible. Besides, a comfortable temperature must be arranged for, in order to prevent shivering (involuntary muscular contraction). The restlessness of small children makes for additional difficulty.

e. Poor contacts

Shifting or loosening of the electrodes or drying up of the electrolyte can completely ruin a tracing.

The modern advance toward the solution of problems of this kind points up the possibility of undertaking electrocardiography during "normal" work.

5. Finally, a number of less essential properties can be mentioned all in one breath, especially sturdiness of construction, ease of operation, reliability, good appearance, light weight and moderate price.

B. Historical Review

Let us briefly consider the instruments which have been used in the course of the years, and how they have developed, adequately to meet the demands made of them.

We can ignore the very old multiplier (forerunner of the galvanometer) which was hardly in a state qualitatively to indicate an electric effect during the heart beat.

The first usable instrument was:

1. The capillary electrometer

This apparatus, invented by Lippmann in Paris in 1873, in its simplest form was a thin U-tube filled with mercury and sulfuric acid and provided with two electrodes (fig. 7).

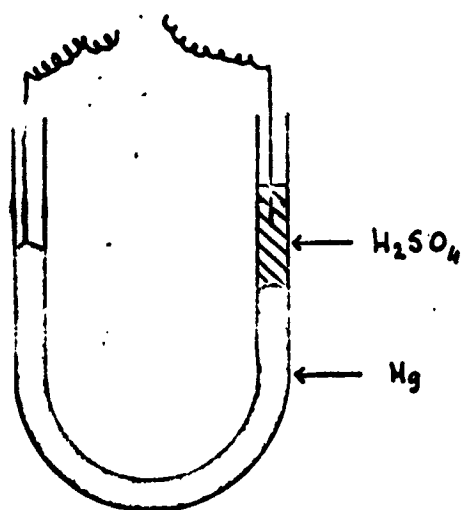


Figure 7. Lippmann's capillary electrometer.

Even a small difference of potential of the electrodes caused by polarization at the boundary layer between the two liquids and the resultant changing surface tension made the mercury meniscus ascend and descend, depending upon the polar state and proportional to the applied voltage.

Internal resistance could be raised to high values. Recording was accomplished by means of a photographic plate.

The performance of this instrument, which meanwhile has been relegated to the museum, was far from ideal because of the great inertia and internal friction of the mercury column, and it would not be mentioned if it had not been the starting point for a revolutionary development.

It remained for Einthoven--a physician!--to study the physical properties of the capillary electrometer by writing and solving the differential equation for the system. He analyzed the recorded tracings mathematically and calculated the "true" EKG. Later subsequent measurements substantiated his results completely.

Dissatisfied, and in search of a more elegant technique of measurement, he arrived at the construction of

## 2. The string galvanometer

This is essentially a 2 to 5  $\mu\text{m}$  thick-silvered or gold-plated quartz thread (the "string") vertically stretched between the pole shoes of an electromagnet.

As known, the effect rests on the Lorentz force: if the voltage to be measured is connected to the two outer ends of the string, then the electric current can flow, forcing the thread sidewise from its position of equilibrium under the effect of the strong magnetic field. The movements were so slight that they had to be observed through a microscope or photographically recorded. The speed, which depends upon the natural frequency of the moving system (hence of the thickness and tension of the string) and upon damping (by the magnetic field) is substantially better than that of the capillary electrometer. Also the sensitivity (likewise determined by the string tension, but in a reversed sense, and by the strength of the magnetic field) can be much increased. Internal resistance is approximately  $10^4 \Omega$ .

A limitation of the speed was set by the excessive thickness of the string, even though it was very thin. (How fine Einthoven's strings were is demonstrated by the following incident. The attempt to make still thinner models had led to failure. One day an accidental beam of light penetrated the laboratory, and it caused a fine glitter: the room was seen to be full of floating quartz threads!)

## 3. Electronic apparatus

Application of electronic devices in electrocardiography constitutes a big advance. It offers the possibility of meeting practically

all the requirements mentioned above, at one time. First may well be noted the greater input impedance (ca.  $10^7 \Omega$ ), which makes the various disagreeable methods of reducing skin resistance unnecessary (page 11). Sensitivity can now be selected at will, as a consequence of the initial unlimited possibility of amplification.

The instruments described above were themselves the indicators.

Electronics, in addition to the amplifier (which formerly was used in combination with the string galvanometer) has furnished still another indicator--which per se is passive--namely the electron beam oscilloscope. The low value of the quotient  $m/e$  for an electron<sup>1</sup> makes this apparatus so fast that /16 the indicating time of the voltmeter as a whole is determined by the properties of the preconnected amplifier.

Finally--and particularly in association with use at the hands of medical personnel--these favorable characteristics must not go unnoticed: simplicity of operation, reliability (no breaking of the thread as the consequence of overvoltage!), no bulk (this last above all since the advent of the transistor).

#### C. Recording technique

In the course of development of instruments, recording technique has a special place of its own. For this reason it must also be discussed separately.

Both the movement of the mercury column of the capillary electrometer and the movement of the thread in Eindhoven's apparatus were recorded on photographic

---

<sup>1</sup>The setting of an oscilloscope is determined by the time required for an electron to pass between the deflecting plates. The speed of travel of an electron, determined from  $1/2mv^2 = eV$ ,  $\frac{m}{e}$  is small.

plates. The oscilloscope also makes use of this method, though optical instrumentation can be simpler. The obsolete plate is replaced today by a roll of sensitized paper.

The use of photography as a recording method has certain drawbacks which must not pass unnoted. First in medical practice is the time required for the finishing process, which is often regarded as a hindrance. Both patient and physician are impatient to see immediate results. This also is true for the experimental physicist, who sometimes cannot determine directly if a specific setting of his apparatus is accurate. In other words, "aiming" at a certain target is not quite possible.

Thus the need was felt for so-called direct recorders. We shall discuss a few.

1. The pen recorder, consisting of a pen with ink reservoir at the end of a rather long arm. Execution of a certain deflection in a short time requires much energy, in view of the rather large mass. Modern amplifiers can adequately meet this requirement, but frequency constitutes a great problem. The natural frequency of the system can be made as high as desired, of course, but the problems arising for example from the deflection of the arm are not solved in this way. In practice the frequency range is set at about 100 cycles.

2. The hot indicator (fig. 8). An electrically heated wire is supported on a rotating drum. Over the drum there passes a strip of black paper with a thin white line of a substance with suitably low melting point. The small contact surface between wire and paper makes it possible to trace the finest, most contrasting line. A special advantage of this recorder is that the deflection is continuously recorded parallel to the drum shaft while the

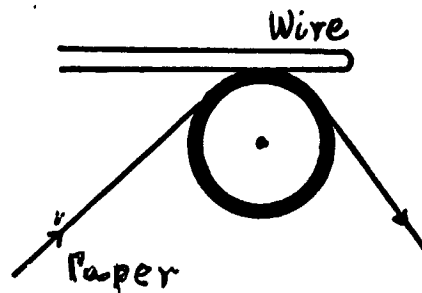


Figure 8. The hot indicator.

pen describes a part of a circle (1 ). For the same reasons as above, the maximum frequency here is approximately 100 cycles.

3. The liquid jet. Ink is projected through a thin bent nozzle as a very thin jet. Because of the slight mass of the nozzle, the natural frequency can be raised to about 800 cycles. An example of such a device is the "Mingo-graph." If the model is a multichannel recorder, a difficulty can arise: the most minute particle of dust in the mouth of the pipe can deflect the liquid jet by a certain angle. If this deflection is in exactly the same direction as the running direction of the paper, or if it is <sup>is</sup> opposed thereto, an apparent difference in time between the channels can be introduced.

4. High-speed photography. Recently much attention has been given to the ultrafast developing of photographic materials. It is now possible to see an image within one minute after recording. A further "development" in this field can be expected.

## CHAPTER 4.

### Placing the Electrodes and EKG

For registration of a difference of potential as a function of time /18 (a so-called lead), two electrodes are placed anywhere on the body surface. In the beginning the possible choice was  $\infty^2$ .<sup>1</sup> It was Einthoven--pioneer of electrocardiography that he was--who was able to bring uniformity to it (page 30). He placed the electrodes on the right arm (R), left arm (L) and left leg (F), which delivered leads I, II and III which are still used today (fig. 9). It will be appreciated that another arrangement is not necessary because the electric current penetrates so little into the extremities that they can be regarded as equipotential volumes.

For the leads of every three electrodes, hence also for the limb leads, Einthoven's summing law is valid:

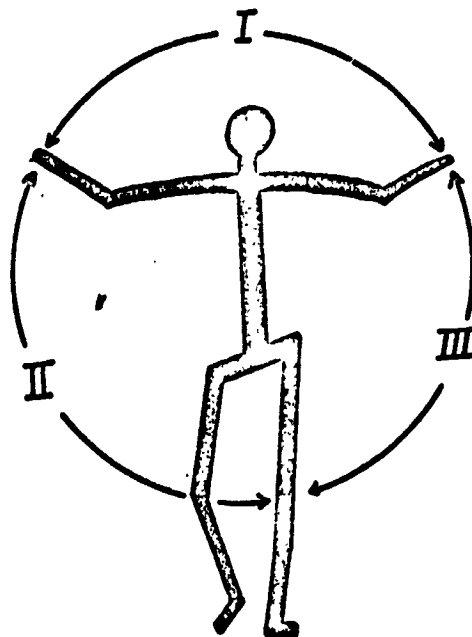
$$I + II + III = (V_L - V_R) + (V_R - V_F) + (V_F - V_L) = 0. \quad ^2) \quad (1)$$

In the introduction it was noted that ~~the~~ answer has been sought to the question which was asked in medical circles, "What about the potentials of /19

---

<sup>1</sup>It might be supposed that the number must be  $\infty^4$ , i.e.,  $\infty^2$  possible choices for each electrode, but it is clear that since we are measuring differences of potential, we can leave it at  $\infty^2$ ! (See also Einthoven's summing law on this page).

<sup>2</sup>In medical literature it is customary to indicate lead II by  $V_F - V_R$ . Thus with  $II^* = -II : I + III = II^*$ .



$$\begin{aligned} \text{I} &= V_L - V_R & \text{II} &= V_R - V_F \\ \text{III} &= V_F - V_L \end{aligned}$$

Figure 9.

each extremity separately?" The fact that the problem--which could be solved up to a certain point--was brought up is not in itself incomprehensible, but the uncertain physical grounds on which work previously was based sounds that way. We know now that the question is meaningless, but many years passed, alas, before the physicist, travelling slowly and hesitantly over unaccustomed terrain, traversed similar wrong routes before he again was able to make connection with the right path.

The desire to be able to measure "a potential" led meanwhile to the introduction of the so-called Central Terminal (C.T.) (Wilson): of the electrodes of the three extremities are connected via three like resistors (the major difficulty was related to skin resistance, which was however minor in comparison to the input impedance of the measuring instrument) with each other

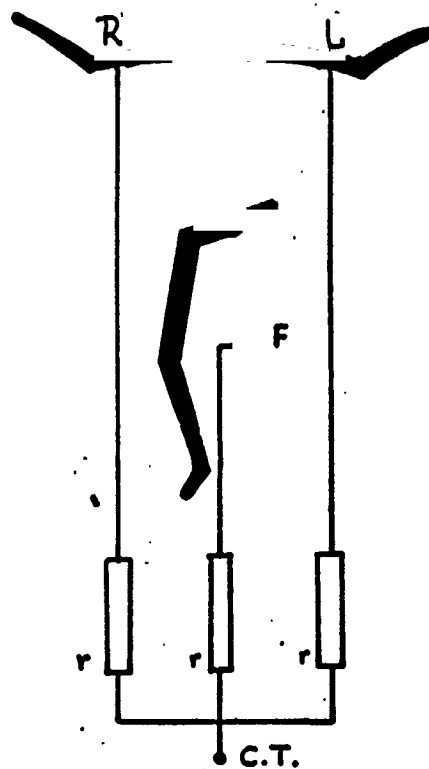


Figure 10. The central terminal.

(fig. 10), then a point is obtained (C.T.) which can serve as a reference point in measurement of voltages in the human body.

In the past it was customary to take zero for "the" potential of the C.T., or to assign a constant potential during the heart beat. Both approaches were inaccurate. It is possible to say that  $V_{CT}$  at every instant is the average of  $V_R$ ,  $V_L$  and  $V_F$  (with reference to any point):

$$\frac{V_R + V_L + V_F}{3} = V_{CT} \quad (2)$$

That this is true can be seen by assuming that the input impedance of the apparatus is so great that no current passes from the C.T. to the apparatus.

In that case, according to Kirchhoff's first law, the equation must be

$$\frac{(V_R - V_{CT})}{r} + \frac{(V_L - V_{CT})}{r} + \frac{(V_F - V_{CT})}{r} = 0. \quad (3)$$

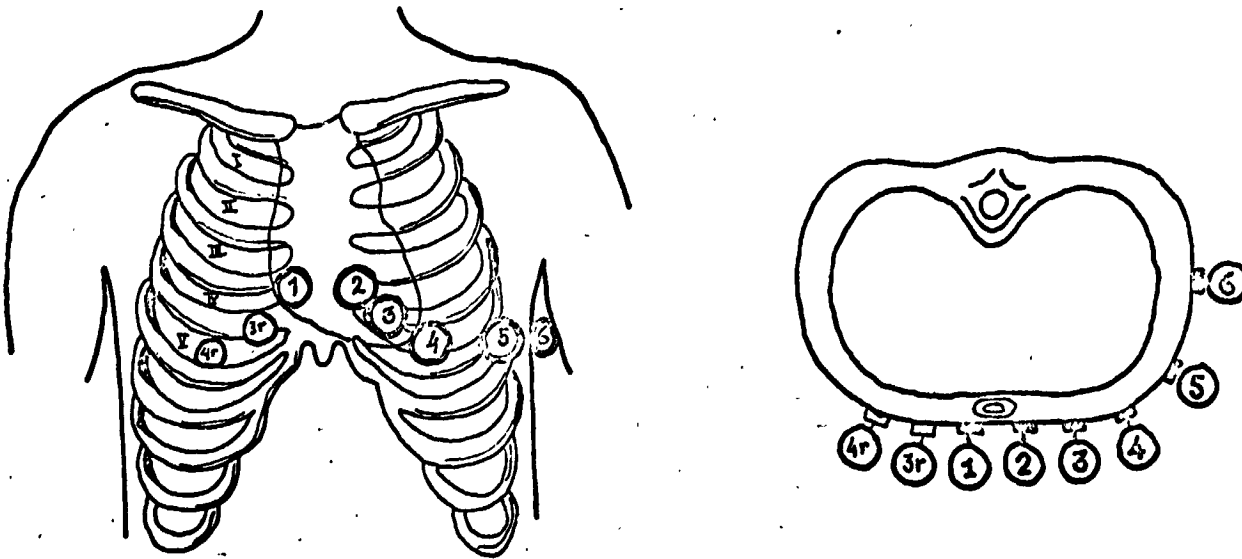


Figure 11. The customary placement of the electrodes.

With reference to the Central Terminal we can measure the voltage at /20 any point or in the body, e.g.,  $V_R - V_{CT}$ . It is easy to see that

$$V_R - V_{CT} = \frac{2}{3} \cdot \frac{(V_R - V_L) + (V_R - V_F)}{2} = \frac{2}{3} \left( V_R - \frac{V_L + V_F}{2} \right) \quad (4)$$

Although nothing is wrong here, it will be recognized that in this way the operation is cumbersome, and that certainly no more information is obtained than when any other part of the body is chosen as reference. More can be learned if more electrodes are applied in addition to the three limb electrodes. Figure 11 shows how the electrodes are usually selected in clinical practice.

The basic form of an electrocardiogram is outlined in figure 12. The internationally accepted indication of the marking points where the extreme values lie, i.e., the letters P, Q, R, S, and T, derive from Einthoven.

In order to be able to make the curve conform to what goes on in the /21 heart, we will briefly review the anatomy. (In Chapter 2 the subject is treated in detail.)

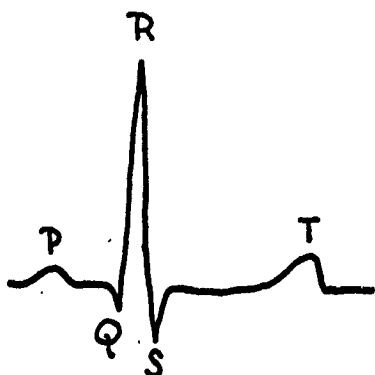


Figure 12. Basic form of EKG.

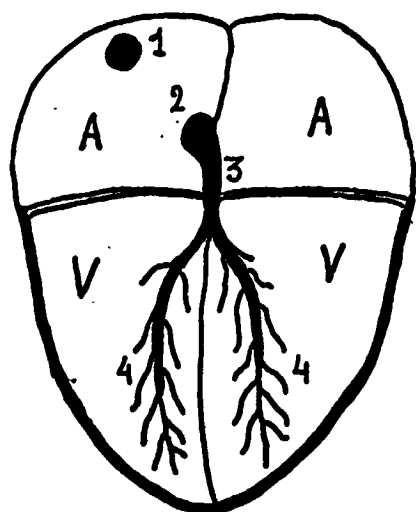


Figure 13. Schematic diagram of the heart. 1, Sinus node; 2, atrioventricular node; 3, bundle of His; 4, branched bundle; A, atrium V, ventricle.

The contraction of the myocardium is not, as in the case of the other muscles, caused by an impulse from the nervous system. The heart (fig. 13) has at its disposal its own pacemaker: the sinus node, a part of the myocardium with a special characteristic undifferentiated structure. A stimulus from this pacemaker can spread over the muscular wall of the auricle in a very short time and cause its contraction. Excitation of the auricle stimulates the so-called atrioventricular node, whence via the bundle of His (which penetrates the non-stimulatable connective tissue wall between the auricle and ventricle) further transmission to the ventricle occurs. In the ventricle the transmitting system bifurcates into two so-called branched bundles from which a finer branching system delivers the stimulus to the myocardium. The muscle tissue itself transmits the stimulus further from cell to cell.

A heart cycle occurs in its totality as follows. First the atria contract, stimulated by the sinus node. With a slight lag in the transmission by the atrioventricular node and the bundle of His, the ventricular contraction takes place (this lag amounts to about 0.20 sec normally, the PQ time of the EKG). The right and left heart valves function synchronously. /22

The following can be said of the origin of the electric effect in the heart. The excited part of the myocardium is separated by a thin boundary layer from the part in which activity is still to occur. Between both sides of the boundary there is a difference of potential so that the potential on the excited side is lower than that on the other (double layer). Thus an electric field is established varying in time and place with the heart beat. (Details on the above will be discussed in Chapter 16.)

The EKG can now be interpreted as follows:

P-top ~ atrium contraction (so-called excitation or depolarization)  
QRS-complex ~ ventricle contraction (excitation or depolarization)  
T-top ~ relaxation of the ventricle, a process of restoration (so-called relaxation or repolarization).

Between T and Q the ventricles change to the rest or polarized state.

How the special EKG forms can be associated with pathological conditions will not be discussed.

## CHAPTER 5.

### The Electric Dipole

At the time that physics began to concern itself with problems of /23 electrocardiography, a fair portion of the applicable theory was clear. Before we come to the actual adaptation, it is necessary to discuss certain concepts in electricity. First of all attention must be paid to the concept of the dipole, which is so important in vectorcardiography.

If we inquire how the physicist describes the phenomena that occur when a current passes through a medium, we immediately encounter Ohm's law. If we omit complexities such as polarization which play no essential part in our considerations, this law then has reference also to electrolytic conduction, such as the conduction that takes place in the human body.

If there is assumed, in analogy to the known conducting wire, the existence of a homogeneous isotropic piece of tissue in the form of a small cylinder (fig. 14) between the outer ends of which there is a difference of potential  $\Delta V$ , it is possible to write, for the current passing through the cylinder

$$I = \frac{\Delta V}{R} \quad (5)$$

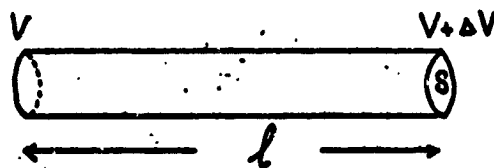


Figure 14.

Or, with  $R = 1/\sigma s$  in which  $\sigma$  is the specific conductivity,  $s$  the surface of the cross section and  $l$  the length of the cylinder

$$I = \frac{\Delta V \sigma s}{l} \quad (6)$$

For a current density  $j$  (= current per surface unit)

$$j = \frac{I}{s} = \frac{\Delta V \sigma}{l} = \sigma E \quad (7)$$

where  $E$  is the electrical field strength.

The expression thus obtained itself very well to generalization, /24 i.e., to a description of more general problems whereby we do not, as above, have to deal with homogeneous potential fields.

Thus, let  $E$ ,  $\sigma$  and hence  $j$  be functions of the location (thus, of  $x$ ,  $y$  and  $z$ ). The expression for current density then becomes a vectoral equation which, when we limit ourselves to isotropic media--in which  $\sigma(x, y, z)$  is a scalar magnitude--can be written as

$$\vec{j} = \sigma \vec{E}. \quad (8)$$

Of course, the human body with all its different kinds of tissues does not meet the condition of isotropy, but we disregard the complications that arise from this situation.

If we now consider a tissue volume  $dx dy dz$  as much electric charge will always flow out as goes in (stationary current), or, stated in a formula

$$\int (\vec{j} \cdot d\vec{O}) = 0. \quad (9)$$

With reference to the Gaussian theorem  $\int (\vec{j} \cdot d\vec{O}) = \int \text{div } \vec{j} \, dV$  (here  $V$  is a volume)

we can write it as

$$\text{div } \vec{j} = \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z} = 0. \quad (10)$$

Combination with the known

relationship:

$$\vec{E} = -\text{grad } V = - \left( \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right) = -\nabla V \quad (11)$$

we have for  $\text{div } \vec{j}$  (for a homogeneous medium)

$$\text{div } \vec{j} = -\sigma \text{div grad } V = -\sigma \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right) = -\sigma \nabla^2 V = 0. \quad (12)$$

or

$$\nabla^2 V = 0 \text{ (Laplace equation)}. \quad (13)$$

What will the formula now be for an element of the myocardium itself?

Since the electric phenomena originate right here, the description of them

with local values must indicate something. It is clear that a single /25

extra term in the expression for  $\vec{j}$  yields exactly the difference sought between

the current density of a volume-element inside and outside the heart. If

we designate the motor field strength  $\vec{E}^*$  and the motor current density  $\vec{j}^*$ , then

$$\text{outside the heart } \vec{j} = \sigma \vec{E} = -\sigma \nabla V \quad (14)$$

$$\text{inside the heart } \vec{j} = \sigma \vec{E} + \sigma \vec{E}^* = -\sigma \nabla V + \vec{j}^* \quad (15)$$

For both cases:  $\text{div } \vec{j} = 0$ .

The extra term  $\sigma \vec{E}^* \equiv \vec{j}^*$  for the myocardium is now precisely the magnitude with which electrocardiography is concerned. It precisely characterizes the electric action of the heart, caused by electrolytic processes in the muscle tissue. The determination of this  $\vec{E}^*$  as function of location and time must yield information concerning normal or abnormal functioning of the heart. It must be immediately added that this magnitude cannot be determined from measurements of differences of potential on the body surface: however, the attempt can be made to determine it as far as possible.

From the above information we shall go further into pure physics, and later, armed with the knowledge thereby obtained, we will return to the heart.

Let us consider a homogeneous isotropic and infinitely extended medium whereto at a determined place P a current I is applied (e.g., with a long wire which is entirely insulated except at the outer end) and again taken off (single pole, fig. 15). The first thing to be noted is that the current divides around the point of introduction in a symmetrical sphere. The current density at distance r from P is then

$$j_r = \frac{I}{4 \pi r^2} \quad (16)$$

or, with  $j_r = \sigma E_r$

$$j_r = \sigma E_r = - \sigma \frac{dV}{dr} = \frac{I}{4 \pi r^2} \quad (17)$$

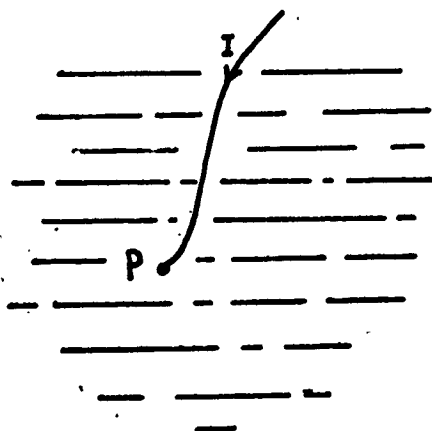


Figure 15. Monopole.

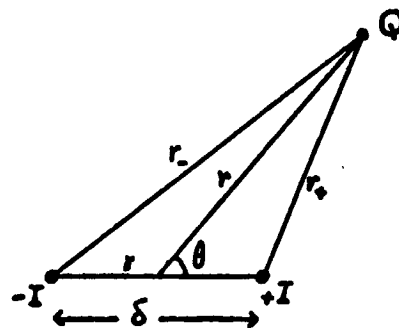


Figure 16. Current doublet.

If we integrate (17) and if we then select as a limiting condition  $V_{\infty} = 0$ , then for the potential at distance  $r$  from  $P$  we have

$$V = \frac{I}{4\pi\sigma r} \quad (18)$$

Let us now imagine not one but two wires inserted in the medium, one of which supplies the current while the other carries it off, in other words two poles, one positive and one negative--a so-called current doublet--, then we may also ask here about the potential at an arbitrary point  $Q$  (fig. 16). We assign  $Q$  by the distance  $r$ , calculated with respect to the middle of connecting line  $\delta$  between the two poles and angle  $\theta$  which  $r$  makes with  $\delta$ . We obtain  $V$  by superposition of the potentials caused by each of the two poles separately, thus:

$$V = \frac{I}{4\pi\sigma} \left( \frac{1}{r_{(+)}} - \frac{1}{r_{(-)}} \right). \quad (19)$$

If we make  $\delta \ll r$ , then we can write for it

$$V = \frac{I \delta \cos \theta}{4\pi\sigma r^2} \quad (20)$$

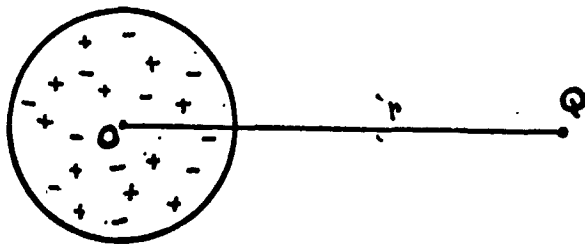


Figure 17. Current poles in conducting medium.

We now let  $\delta$  approach zero and at the same time let  $I$  approach infinity while we hold the product  $I\delta \equiv D_i$  constant, and we then obtain a dipole as limit and we can describe the field by

$$V = \frac{D_i \cos \theta}{4\pi\epsilon r^2} \quad (21)$$

In the description of the electric phenomena of the heart we shall see over and over again what an important place the dipole concept occupies, and for that reason we shall go more deeply into it and approach the problem from two other angles.

a. We imagine a spherical part of a homogeneous infinitely extended conducting medium. Inside this part we assume an arbitrary number of poles to be present, both positive and negative, so that we know the currents that come into the medium and leave it as well as the coordinates of each pole, in one system of coordinates, of which we select the origin in the middle of the sphere.

Again we ask about potential  $V$  at a point  $Q$  outside the sphere, at 27 distance  $r$  from the center (fig. 17). In principle we can of course obtain  $V$  through the potential caused by each pole, writing and summing for all poles.

Something essentially new does not emerge from the calculation, of course. We could more profitably try to develop  $V$  in a power series in  $r$  (see also page 123).

Since the choice of an infinitely extended medium also permits  $r = \infty$  and the series naturally must converge, these negative powers of  $r$  must be included. So we can write

$$V = \frac{\phi_1}{r} + \frac{\phi_2}{r^2} + \frac{\phi_3}{r^3} + \dots + \frac{\phi_i}{r^i} + \dots \quad (22)$$

(only valid outside the sphere!) wherein  $\phi_i$  is a function of the direction of the radius vector at  $Q$  and must be such that  $\nabla^2 V = 0$  (see <sup>15</sup>Chapter 15 page 116).

Now the field of a single pole is described by  $V = I/4\pi\sigma r$ . In other words, the first term is none other than the potential of the total single pole formed by all poles together. Since  $V$  must satisfy the Laplace equation  $\nabla^2 V = 0$  and we assumed above that in the totality no current goes in or out of the sphere, this term in our case is zero. By analogy we see that the second term represents the potential of the whole dipole while the following terms represent poles of higher order. The requirement for practical applicability of this representation is naturally that the series converge rapidly.

b. Finally we treat a simple and completely calculable case. We consider a homogeneous conducting and infinitely extended space. In a spherical part thereof (radius  $a$ ) there is a known constant homogeneous motor field strength  $\vec{E}^*$  acting in  $x$  direction (fig. 18). (The coordinate system is selected as under a.)

We designate the part of the space inside the sphere as 1, the part 28 outside as 2. In analogy to what has been said above (page 25)

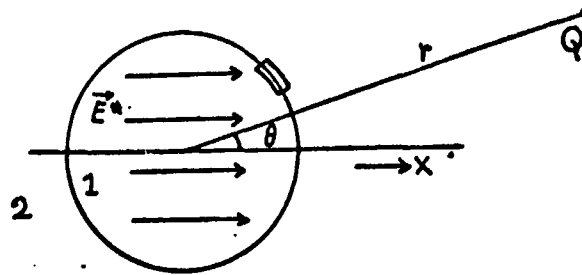


Figure 18. Homogeneous motor field strength.

$$\vec{j} = -\sigma \nabla V + \sigma \vec{E}^* \quad (15)$$

is valid

with  $\vec{E}^* = \text{constant}$   $r \leq a$

and  $\vec{E}^* = 0$  for  $r > a$

and  $\text{div } \vec{j} = 0$  for all  $r$ , with limit condition for  $r = a$ .

We should now find an expression for  $V_1$  and  $V_2$ , both as function of  $r$  and  $\theta$ . For  $V_1$  we expect an equation with a positive power of  $r$ , so that within the sphere the potential can remain finite, while in the formula for  $V_2$ ,  $r$  will appear with a negative power, that for  $r \rightarrow \infty$ ,  $V \rightarrow 0$ . Moreover in both cases  $\nabla^2 V = 0$  must be satisfied. It appears then that we can write:

$$V_1 = Ar \cos \theta = Ax \quad (23)$$

and

$$V_2 = \frac{B \cos \theta}{r^2} \quad (24)$$

wherein  $A$  and  $B$  are constants, i.e., independent of the coordinates  $r$  and  $\theta$ .

Equations (23) and (24) (of which the latter represents the potential as result of a dipole) will be justified in the following calculation.

We determine  $A$  and  $B$  by means of the limiting conditions:

1. The potential at the surface of the sphere (thus of  $r = a$ ) must be continuous, i.e., we must be able to describe  $V$  with the equation for  $V_2$  as well as with the equation for  $V_1$ . Since the potential must have a unique value,

$$A a \cos \theta = B \frac{\cos \theta}{a^2} \quad (25)$$

from which it follows that  $B = a^3 A$ .

2. For the second limiting condition we consider a thin box-shaped 29 element that is located half in the sphere and half in space 2. For this element the entering current is the same as the outgoing current ( $\text{div } \vec{j} = 0$ ). Thus the normal component of current density, calculated from 1 and also from 2 must yield the same result. Thereby we obtain a second relation between  $A$  and  $B$ .

$$\begin{aligned} & -\sigma \text{grad } V_1 + \sigma E^* \cos \theta = -\sigma \text{grad } V_2. \\ \text{of} \quad & -A \sigma \cos \theta + \sigma E^* \cos \theta = 2 B \sigma \frac{\cos \theta}{a^3} \end{aligned} \quad (26)$$

Combination of (25) and (26) yields for  $V_2$ :

$$\begin{aligned} V_2 &= \frac{E^* \cos \theta a^3}{3 r^2} = \frac{E^* \cdot 4/3 \cdot \pi a^3 \cos \theta}{4/3 \cdot \pi 3 r^2} \\ \text{hence;} \quad V_2 &= \frac{E^* \cdot \text{volume} \cdot \cos \theta}{4 \pi r^2} \end{aligned} \quad (27)$$

If we now write:  $E^* \cdot \text{volume} = D_v$  and  $D_v = D_1 / \sigma$ , we get exactly the expression that we calculated for the potential as result of a current dipole (page 26 (21)).  
(27) is then the expression for a voltage dipole.

From this simplified picture it appears that the terms of higher order in the general series progression (see a) are lacking here.

Let us assume finally that  $E^*$  is a function of the coordinates. Then clearly (27) goes over into an integral form:

$$V_2 = \frac{\int_{\text{vol}} E^* (\vec{r}) \cos \theta \, d(\text{vol})}{4 \pi r^2} \quad (28)$$

Here the form of the volume is arbitrary.

## CHAPTER 6.

### Einthoven's Triangle

Before we fit the description of current fields to the human body /30 (or models thereof) we will first consider what had been going on in the world of medicine independently of the physical theories we have been discussing.

We start with a train of ideas that originates with Einthoven,<sup>1</sup> a set of ideas which in spite of limitations and deficiencies--which Einthoven himself fully recognized--had attracted much attention and which can be regarded as the basis of modern vectorcardiography.

Einthoven's reasoning ran more or less like this: given, that we overlook the fact that the heart is not located exactly in the middle of the body, let us further assume that the human body is completely symmetrical (inside and out), and finally let us assume that we can represent the electric activity of the heart as a directional value (fig. 19)--designated by Einthoven as "manifest potential difference" or "manifest value," and subsequently designated "heart vector." If now at a specific instant this vector is directed along

---

<sup>1</sup>Einthoven performed pioneer work in the field of electrocardiography, his achievement being crowned by the Nobel prize (1924). The uniformity that he brought into the discipline also testifies to his importance. We may mention here: sensitivity (1 millivolt = 10 mm, page 11), speed of travel of the paper 25 mm/sec, page 12), localizing of the electrodes on the extremities (page 18) and the expressions P, Q, R, S and T in the EKG (page 20).

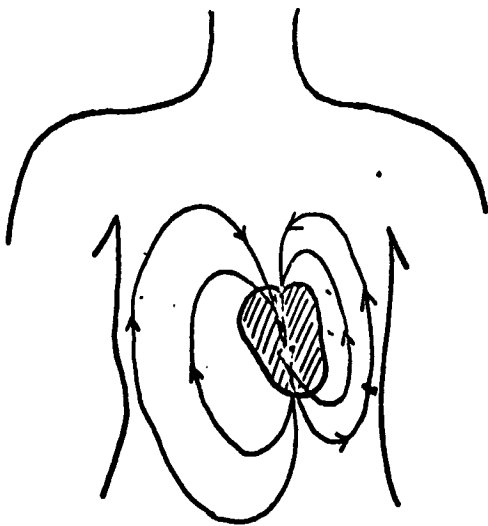


Figure 19. "Instantaneous picture" of the heart vector.

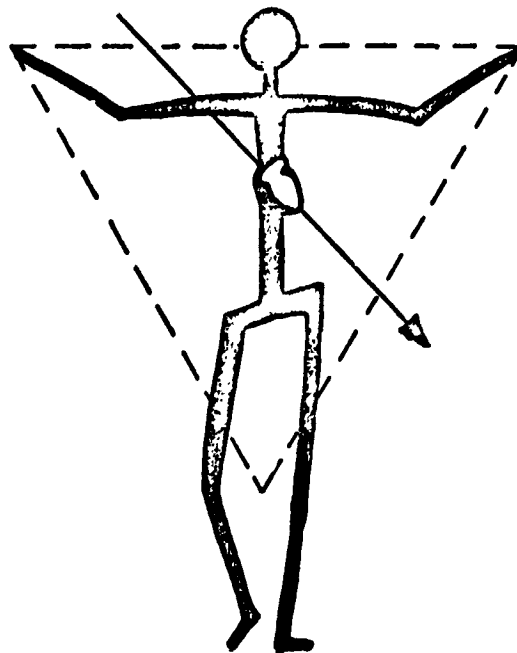


Figure 20. Einthoven's triangle.

the long axis of the body, there will be no difference of potential between the right and left arm. When on the contrary the vector makes an angle with "the vertical," then the horizontal component of one of the two arms will be positive with respect to the other.

This view which yields a relation between the magnitude and direction of the electric activity of the heart and the difference in potential between two points on the body (for a specific moment!) can be generalized. /31 Einthoven accomplished this by means of an equilateral triangle, of which one of the sides was represented by a connecting line RL between right and left arm, while the third angle was below the lower back or, if desired, at the usual location for the foot electrode (fig. 20). With a certain approximation the position of the "arrow" from the three sides of this "Einthoven triangle" was then a measure of the difference of potential between the extremities (fig. 21).

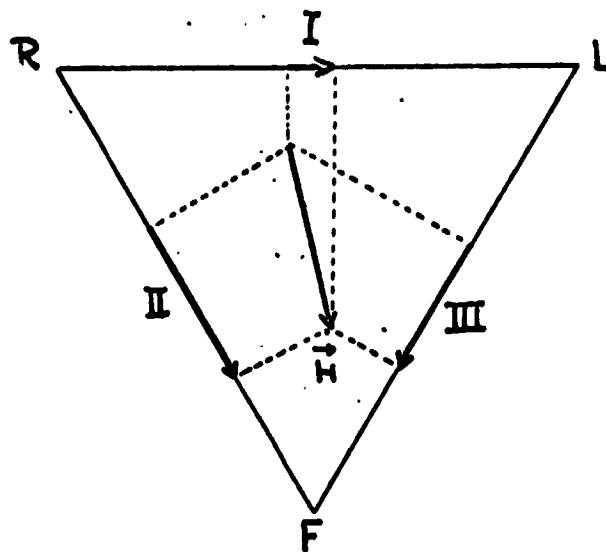


Figure 21. Projections from the sides  
of the triangle are of course not vectors!

The idea gained general acceptance because of the simple and striking 32 relationship between the limb leads and what could be conceived concerning heart activity. Later we shall see that acceptance of the idea in this special form cannot be sustained. Even at this stage, we cite an argument that was considered to be valid at the time.

A proposition from plane geometry states that if the projections of a line on the sides of an equilateral triangle, furnished with the appropriate sign, are added, the sum is zero. A more elegant proof of the exactness of Einthoven's insight would be hard to find, because if we accept this proposition we have complete agreement with the rule that the difference of potential between R, L and F must be exactly zero (page 18)! Proceeding from subsequent observations, we already see how this (apparent) argument begins to weaken when we consider that in actuality the heart is not positioned geometrically at the center, nor is the body symmetrical, and that moreover there is no valid

reason for making the sides of the triangle exactly alike. A glance at figure 20 convinces us of the unreality of the notion that the association between heart vector and leads depends upon the length of the arms!

Einthoven's thought developed along various lines (sometimes in error). We shall mention instances from the past showing in more detail that this development has led us step by step to present-day vectorcardiography.

In the first place there is the adaptation of instruments. The two-dimensional surface on which the heart vector  $\vec{H}$  is assumed to move, made it possible to determine it directly from leads I, II and III. A simple calculation from plane geometry shows that the y component (a system of coordinates is introduced as in figure 22) to be determined is expressed as

$$y = \frac{2}{\sqrt{3}} \left( V_F - \frac{V_R + V_L}{2} \right) \quad (29)$$

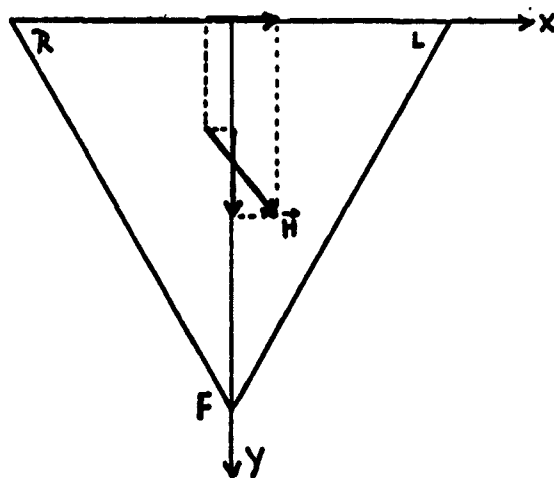


Figure 22. Determination of x and y components (caption is incomplete:  $T_F$ ).

while the x component is immediately represented by  $V_L$  and  $V_R$ . In other 33 words, if R and L electrodes are connected to a point Q via like resistors, the difference of potential between F and Q immediately yields the y component in the form of a voltage, leaving out factor  $2/\sqrt{3}$  (which it is obviously natural to introduce in electronics). If this voltage is connected to one pair of plates of a cathode ray tube and  $V_L - V_R$  are connected to the other pair, then during a heart cycle a curve can be described which corresponds to the two-dimensional course which the point of the arrow traces, representing the heart vector. By periodic depression of the electron beam, a time marking can be introduced (see page 37). In this way, after many years, a kind of vector-cardiogram (VKG) could be recorded.

How far the acceptance of the (in itself valuable) concept of the triangle can lead in the direction of absurdity is illustrated by the fact that a cathode tube has been constructed with three paired plates at angles of  $60^\circ$ , to make a VKG directly from leads I, II and III. This no doubt expensive apparatus was apparently preferred to the use of a couple of resistors and an ordinary tube!

The abandonment of the flat surface was a great step forward. With addition of the fourth electrode, e.g., on the back, (W) the Einthoven triangle could be expanded to an equilateral tetrahedron (Wilson)(fig. 23).

Here also, mistakenly, the assumption was that the projection of the heart vector on one of the ribs would yield the difference of potential between the electrodes, coinciding with the angular points connected by the ribs.

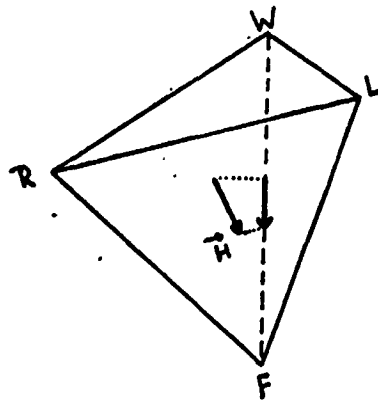


Figure 23. Equilateral tetrahedron

W = Wilson.



Figure 24. Stereoscopic representation of  
the three-dimensional curve.

The idea that the electric heart activity was not a two-dimensional but rather a three-dimensional phenomenon was accepted, however slowly. In 1934 representation with the heart vector this meant that "the arrow point" had to be described not as a flat, but as a three-dimensional curve (fig. 24). To obtain this curve the heart vector had to be projected on three mutually perpendicular surfaces. To construct this curve, it is necessary to project

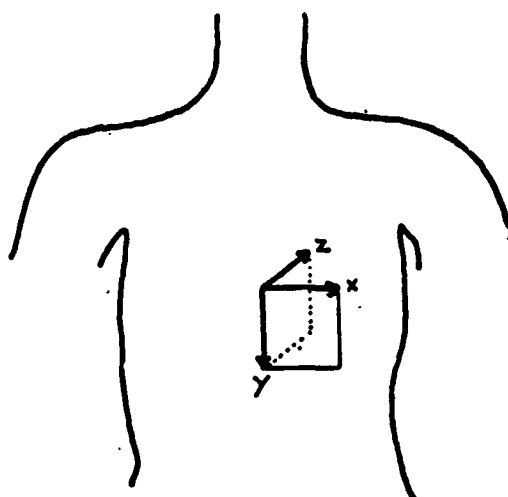


Figure 25. System of coordinates  
in the human body.

the heart vector on three mutually perpendicular surfaces. For these there were chosen the frontal, the horizontal and the sagittal surface or respectively the X, Y, the X, Z and the Y, Z surfaces (fig. 25). In practice this comes near to determination of each of the components X, Y and Z of the heart vector  $\vec{H}$  as a function of time from the differences potential measured on the body surface. By this means <sup>each</sup> of the three projections can then be thrown on an oscilloscope screen and photographically determined (fig. 26).

Originally the attempt was made to select the locations of the electrodes in such a way that each pair separately would yield one component. It is clear that such a localization (so-called cubic system) in the nature of things was unattainable. Opinions on the matter were then very divided, and besides there was chaos concerning the field of vectorcardiography.

We will not review all the endeavors to place the electrodes in such 35 a way that there would be no calculations to handle, so that the components of

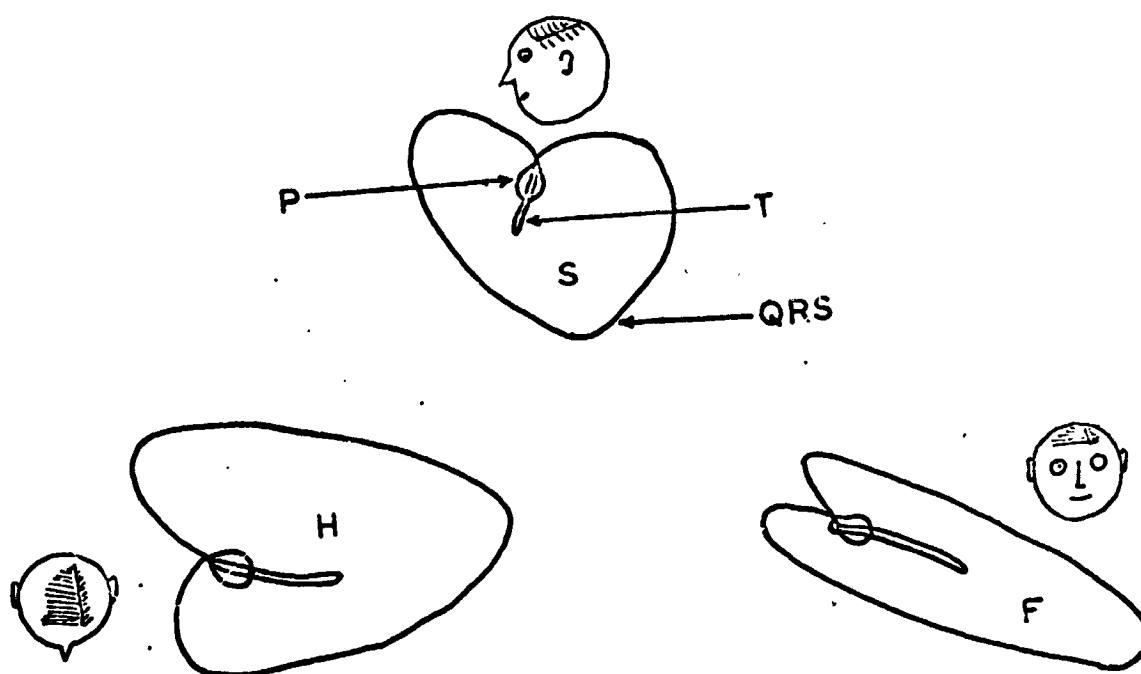


Figure 26. A VKG (see also fig. 81, page 140).

F, Frontal projection (X, Y); H, horizontal projection (X, Z); S, sagittal projection (Y, Z).

the heart vector could be measured directly on the body surface. The systems thus developed were doomed to failure because they were based on intuition and not on physics.

It should be noted that in the course of years many systems were conceived and even used which all produced data for use in their own way. Alas, the vector cardiograms based on them are hardly comparable, and in fact it is possible to call it deplorable confusion.

This serves to bring up the question, if it is actually worth the trouble to work toward the true physical development of vectorcardiography with the hope of achieving some uniformity. It seems well at this point to probe the matter somewhat more deeply.

We might start with the pro-vectorcardiography argument.

/36

1. Vectorcardiography yields more information than electrocardiography.

At first glance this seems to be a false premise since just as in electrocardiography, differences are measured on the body surface in vectorcardiography. And why should one combination of leads impart more information than another? Naturally there is essentially no question here either: there is a fundamental difference in the way in which the two methods apply the information! While it is customary in electrocardiography to record the leads in succession, of necessity it is simultaneous in vectorcardiography so that a VKG essentially is a statement of phase difference. That these phases also contain information can be illustrated by the fact that in some cases a diagnosis can be made which is based exclusively on the phase difference (we will return to this). It is true that often in electrocardiography also a number of leads are registered simultaneously but there is still uncertainty concerning the phase, considering that no single instrument is 100 percent reliable with respect to its time shift. For that matter, it is possible to extract little information from them because the time resolution is too small.

An advantage which is not essential of course but which is nonetheless real in practice is the way the information is achieved, namely in that in vectorcardiography it is customary to work with somewhat greater sensitivity.

2.a. Information is obtained in a more congenial form in many respects. In judging vectorcardiograms (fig. 26) we use to a considerable degree our everyday ability to interpret certain shapes. We recognize our colleague "in the wink of an eye": the difference between a circle and an ellipse is

immediately appreciated. We make use of this faculty in reading electrocardiograms too, although practical experience indicates that the reading is learned with more difficulty and more rapidly forgotten.

b. The total electric event during an entire heart cycle is re- /37  
produced and recorded by the closed circuit which is described by the dot on the oscilloscope screen. The electrocardiogram is "rolled up" as it were, so that the datum occupies a minimum of space.

c. The vectorcardiogram corresponds more closely to actual anatomy than the EKG does. The transmission of electricity and the position of the heart: these are simple concepts that we can imagine, and that we recover in the clear arrangement of the record. It is conceivable that we feel more at home in this case than with the specific physical method of reproduction of a magnitude that varies in time, whereby every direct correlation with reality is lacking.

For the sake of fairness, some drawbacks must also be noted. In the first place, the time element is practically eliminated in a vectorcardiogram. This deficiency is partially overcome by depression of the electron beam of the oscilloscope at constant known time intervals, so that a punctate image appears on the screen. It is possible to get an impression of the speed of advance (see fig. 24) of the dot in each phase of the heart beat from the record. The dot's intensity does not drop abruptly, but rather it gradually diminishes, so that at the end of each strip a point appears (  ) from which the direction of travel, which can be of great interest, can be determined (see fig. 61, page /106).

It is also sometimes difficult to see the transition from S to T in a vectorcardiogram (loop designated as the S-T segment).

Finally there is the intricate mathematical formulation which sometimes arouses resistance in the medical world.

If this barrier can be penetrated, vectorcardiography is surely a step in the right direction, and we advance toward as complete a knowledge as is possible of the human heart.

## CHAPTER 7.

Connection Between Heart Vector and Leads. The Vectorcardiogram.

/38

After consideration of developments in medicine and physics independently of each other, we shall now attempt to tie the two together.

In the trunk of the body the electric current satisfied a differential equation (Chapter 5): thereby we have limiting conditions for the body surface and for the immediate vicinity of the source. It is thus clear that, with an exact physical treatment of what occurs in the human body with reference to electric activity of the heart, we confront a complicated problem. A simplification is in order.

1. All phenomena of which we attempt to discover the background are linear. This is suggested by an equation such as  $\text{div } \vec{j} = 0$ , an equation of the first order, thus without products or squares, of the linear relationship between current density and field strength (Ohm's law) or of the limiting condition  $\vec{j}_n = 0$ . We will make use of this linearity at the end.

2. We assume all electric phenomena to be caused by a stationary dipple (for definition see page /26). This approximation will only be applied provisionally, to be subjected to more rigorous treatment farther on. (We use "dipole approximation," the expression where others use "dipole hypothesis." Since the word "hypothesis" is out of place here, we depart from usual terminology.)

In principle there are three methods whereby we can consider the connection between the heart vector and the observed phenomena:

a. By means of formulas

We select a simple exercise, to represent the human body as a homogeneous isotropic medium with a spherical outline. If we assume the dipole of /39 the heart to be in the center thereof, we can calculate without difficulty the distribution of potential on the surface (Chapter 5). Vice versa, we can come to know something about the source from this distribution of potential (page /25, however).

If we take more complicated cases, we can extend our formalizing. Thus we can gain unique insight into the material, but it is clear that little can be expected from this in practice, since the actual problem is too complex.

b. The digital method

We can attempt to solve the differential equation with limiting conditions by numerical calculation. We will not go into this any further.

c. The analog method

Hereby we make use of a model: a system that is as accurate an imitation as possible. We can now choose or vary a number of magnitudes (actually an unknown number) in order to be able to follow them by measurement of their effect upon our model system. The knowledge gained in this way can then later be applied to the solution of the actual problem.

In our case the model will be a more or less accurate copy of the human body: a cavity filled with an electrolyte, having an artificial heart.

What we will measure is the distribution of potential over the surface as a function of magnitude in the direction of the heart vector. Following our above working hypothesis, we use for our artificial heart an electric doublet (thereby approximating the dipole) which is so constructed--(we will

return to this, page 43)--that orientation and pole strength can be suitably selected.

Before we go into practical details, we will first formulate the problem more in detail, by solution of which the model can help us.

If we assign the three components of the heart vector (in our model the dipole vector) as  $X(t)$ ,  $Y(t)$  and  $Z(t)$ , because of the linear relationship (page 38) of the difference of potential between any two electrodes  $P$  and  $P_0$ , we can write 40

$$V_P - V_{P_0} = a(\xi, \eta)X(t) + b(\xi, \eta)Y(t) + c(\xi, \eta)Z(t) = V(\xi, \eta, t). \quad (30)$$

Here  $a$ ,  $b$  and  $c$  are "constants," dependent only upon the location of  $P$  (expressed in body coordinates  $\xi$  and  $\eta$ ) and not upon time;  $P_0$  is a fixed reference point. This expression, which gives us the relationship between one specific lead and the heart vector with its three time-dependent components and which is valid independently of the properties of the model, can also be generally stated

$$\begin{aligned} V(\xi, \eta, t) = & c_1(\xi, \eta)f_1(t) + c_2(\xi, \eta)f_2(t) \\ & + c_3(\xi, \eta)f_3(t) + c_4(\xi, \eta)f_4(t) + \dots \end{aligned} \quad (31)$$

Thus a linear combination with a plurality of time functions. The dipole approximation permits us with the first three terms to assume that the dipole vector has only three components. For a more exact description however, as shown later (Chapter 15) we shall need to use higher terms.

For each of the  $\infty^2$  electrocardiograms that we can make for one subject, an equation of this kind can be written. Let us restrict ourselves to

a minimum of three (independent) leads: we thus get three equations of this kind, e.g., (in simplified notation)

$$\begin{aligned} V_{P_1} - V_{P_0} &= V_1 = a_1X + b_1Y + c_1Z. \\ V_{P_2} - V_{P_0} &= V_2 = a_2X + b_2Y + c_2Z. \\ V_{P_3} - V_{P_0} &= V_3 = a_3X + b_3Y + c_3Z. \end{aligned} \quad (32)$$

If we now wish to know the nine constants  $a_1, b_1$  and  $c_1$  we can solve for  $X, Y$  and  $Z$  from the potential differences determined for each instant. The equations are

$$\begin{aligned} X &= \alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3. \\ Y &= \beta_1 V_1 + \beta_2 V_2 + \beta_3 V_3. \\ Z &= \gamma_1 V_1 + \gamma_2 V_2 + \gamma_3 V_3. \end{aligned} \quad (33)$$

in which  $\alpha_j, \beta_j$  and  $\gamma_j$  are known functions of  $a_1, b_1$  and  $c_1$ . In other words, <sup>41</sup> we can calculate the behavior of the heart vector during an entire heart cycle.<sup>1</sup>

This is the model that offers us the possibility of binding the coefficients  $a_1, b_1$  and  $c_1$  by approximation. For this we orient the artificial heart so the dipole vector is directed along one of the axes of the coordinate system

---

<sup>1</sup>As known, three equations with three unknowns are only soluble as the determinant  $|\alpha_j \beta_j \gamma_j| \neq 0$ . In practice this means that the said determinant must not be too small.

that we have assumed in the body or in the model (see fig. 25, page 34). If we select for this direction e.g., the X axis, the Y and Z will be reduced to zero in equation (33)

$$V_1 = a_1 X. \quad (34)$$

We can solve for  $a_1$  from this.  $V_1$  is derived from one measurement, since we can select X arbitrarily, at least to a degree. Analogously we determine values for  $b_1$  and  $c_1$ .

The fact that coefficients  $a_1$ ,  $b_1$  and  $c_1$  are dependent upon the locations of the electrodes implies that the properties of the model are of major importance. Care must therefore be taken in construction of the phantom. We limit ourselves here to a few general remarks with special reference to the model used here in the laboratory (fig. 27).

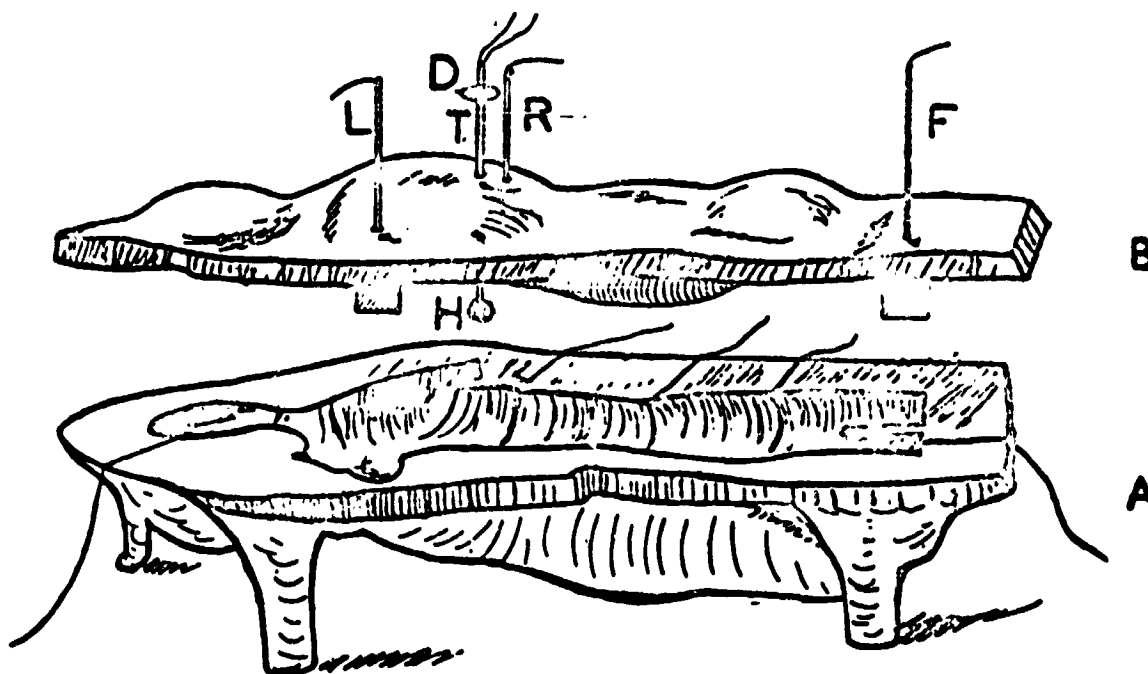


Figure 27. The model.

"Mixed plastic" (insulating material) is used to make a hollow form of a man lying on his stomach, in 1/3 natural size. The interior is accessible through a slit parallel to the frontal surface. The artificial heart can be placed inside through an aperture in the back. The cavity is filled with a copper sulfate solution. The natural inhomogeneities are of course not completely imitated, and we can do no more than simulate the vertebral column and lungs with very poorly conducting parts in the immediate vicinity of the heart.

Obviously all these arrangements cannot be made arbitrarily, but /42 rather they must be based on measurements made on the human body. Without going into the technicalities of measurement, we simply note that for the specific conductivity of "normal" muscle a value of 200 to 300  $\Omega/\text{cm}$  (longitudinal measurement) was determined, and that bone is a very poor conductor, while for the specific resistance of the lungs a magnitude approximately four times that of the surrounding tissue was determined.

In the model we supplied a cork spinal column. The lungs were simulated satisfactorily with two modelled sand bags.

Concerning the latter a few more remarks can be made. It is obvious that the relationship of resistance between "lungs" and electrolyte was independent of the concentration of copper sulfate solution. Perhaps it is less obvious that this relationship is not affected by the absolute size of the grains of sand either. If the grains are larger, then the interstices between /43 them are larger and total conductivity remains unchanged. The law of frequency no doubt has an influence. This law governs the distribution of mass, for through it the space between the larger grains and the smaller is determined.

How much the model still leaves to be desired in the way of authenticity is indicated by the fact that no account is taken of the important anisotropy of muscle (a striated muscle has a much greater specific conductivity in the longitudinal direction than in the direction perpendicular thereto).

The model has neither arms nor legs. The electric current emanating from the heart does not penetrate into these members so that--as already noted (page 18)--the extremities cannot be considered as equipotential volumes.

Since opinions concerning magnitude and effect of inhomogeneities in the body are not always unanimous, there is in addition to the above described standpoint, still another with respect to phantom design. The premise is that a hollow form filled with electrolyte alone (hence no lungs etc.) also yields a meaningful approximation of reality--with or without realizing that more damage is being done than necessary. According to the holders of this opinion, the object of the search is not the dipole whereby the electric activity of the heart can best be characterized, but rather an effective dipole. An effective dipole is understood to be the dipole that we should establish for the homogeneous model, to bring measurements made thereon into conformity with those made on the human body.

With all the shortcomings inherent in these models, there is at hand the human body itself which may be used as a model, in other words--measurement on cadavers. However great the advantages may be, there are still many sources of error, among them those that stem from postmortem changes. We note only that many experiments have been done in this area, not always unsuccessfully.

A few remarks may be made with reference to the artificial heart, the dipole generator.

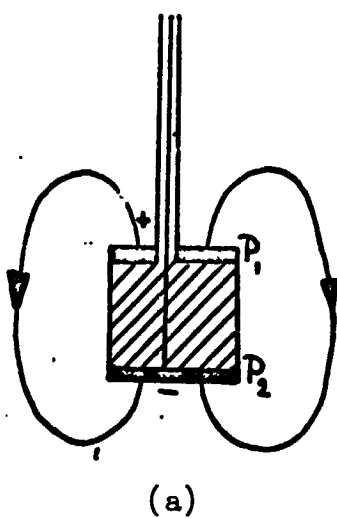
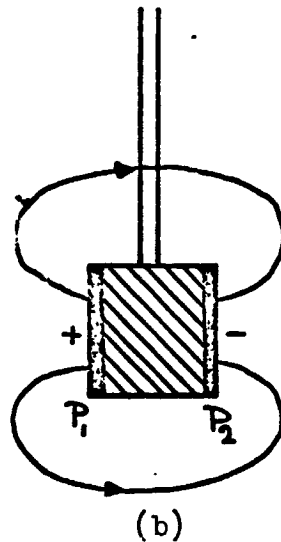


Figure 28.



(b)

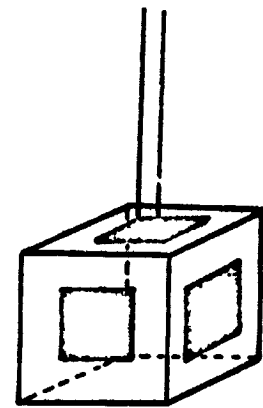


Figure 29.

Two possible forms of the artificial heart.

Structure is as follows: two circular copper plates  $P_1$  and  $P_2$ , each about 2 cm in diameter, are mounted on a cylinder 2 cm long, made of insulating material (fig. 28). By means of 4 volt storage batteries a predetermined difference of potential can be established between the two plates (voltage dipole). A current will then flow in the liquid along known lines of conduction.

/44

The dipole of figure 28a only permits an orientation in the Y direction, but the artificial heart of figure 28b makes X and Z directions accessible also. Use is simpler in the embodiment of figure 29. The poles here are attached on the six surfaces of a cube oriented along the coordinates system. Component X, Y or Z is selected by connection to the appropriate paired poles.

The question may be asked, what kind of dipole should be selected for the generator. There are three kinds, load, current and voltage dipole. It should be evident that the first type cannot be used for practical purposes, since in a conducting medium such as an electrolyte it is impossible to introduce the conductor of a charge which can be applied as desired. The

choice between voltage and current dipole remains. We restrict ourselves to a few remarks on the relative value because it appears that the two types are identical.

Up to the present, measurements of a, b and c have always been relative measurements, really. Naturally it is possible to assign absolute values, by means of the equation

$$V_1 = -a_1 X. \quad (34)$$

to determine absolute values for  $V_1$  and  $X$ .

$V_1$  can naturally be read directly from a voltmeter, but  $X$  must be determined by calculation. Concerning the latter we recall what was said above 45 with respect to the mathematical expression for the dipole (Chapter 5).

If we take a voltage dipole as our starting point, we can write for the potential  $Q$  at distance  $r$  (calculated from a constant)

$$V = \frac{D_v \cos \theta}{4 \pi r^2} \quad (35)$$

(see also (21) page 26 and page 29). This expression is only valid, as we observed elsewhere (page 26) for a pure dipole (mutual distances of the poles  $\rightarrow 0$ , pole strength  $\rightarrow \infty$  and  $D_v = \text{constant}$ ). If in practice we attempt to approximate this ideal, we must apply a very high voltage between the smallest possible poles. But then such great current densities will be developed in the immediate vicinity of the poles that numerous complications will arise for example heating of the electrolyte and consequent sudden lessening of the specific resistance. For this reason we prefer a doublet for our artificial heart: two metallic plates (diameter  $2a$ ) mutually separated by

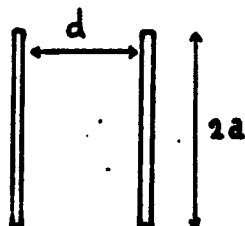


Figure 30. According to Maxwell's formula.

distance  $d$  (fig. 30). The expression for the dipole moment, already calculated by Maxwell, exists for such a system (in which there is no insulating material between the plates)

$$D_v = \Delta V \pi a^2 f\left(\frac{d}{a}\right) \text{ Volt.m}^2 . \quad (36)$$

(We will not discuss in detail the known form of  $f$  in this equation.) Hereby the absolute magnitude of the dipole vector, or of the "heart vector" of the used artificial heart, determined. By making the plates adjustable with respect to each other, the size of the vector can be varied within certain limits. It should be noted that all this is reported only to show how absolute values for  $a$ ,  $b$  and  $c$  may be obtained. In practice these are relative. In vectorcardiography it is more the form of the spatial curve that is considered to be important rather than absolute size. It is true that occasionally work has been done on relative size<sup>1</sup> but clinically it is not very important.

---

<sup>1</sup>For normal use, work is performed with a specific apparatus sensitivity. If the loop for an individual patient is so large that it is no longer shown completely on the screen, the sensitivity can be reduced to 0.7 of normal.

The association mentioned above between leads and heart vector is /46  
the natural basis of the instrumentation. We will consider it briefly.

The starting point of course is the three relationships (page /40) which  
the three components  $X(t)$ ,  $Y(t)$  and  $Z(t)$  of the heart vector yield as  
linear combination of the used leads  $V_i(t)$

$$\begin{aligned} X(t) &= \alpha_1 V_1(t) + \alpha_2 V_2(t) + \alpha_3 V_3(t) \\ Y(t) &= \beta_1 V_1(t) + \beta_2 V_2(t) + \beta_3 V_3(t) \\ Z(t) &= \gamma_1 V_1(t) + \gamma_2 V_2(t) + \gamma_3 V_3(t) \end{aligned} \quad (33)$$

On the preceding page we explained how magnitudes  $\alpha_1 \dots \gamma_3$  could be  
determined by model experiments. We shall assume these 9 coefficients to  
be known in the following.

It is clear--we already indicated it (page /33)--that voltages  $X(t)$ ,  $Y(t)$   
and  $Z(t)$  when they are accurately applied to the deflecting plates of  
oscilloscopes, make it possible for us to make the three projections of the  
vectorcardiogram visible. We shall discuss how we can realize it experimentally.

As an example, let us assume that the three independent leads  $V_1$ ,  $V_2$  and  
 $V_3$  correspond to the potential differences which were determined between four  
electrodes for which we select the earlier mentioned leads R, L, F and B on the  
chest. We should then be able to write (33). (It seems to be a practical  
advantage to take the electrode on the right arm as the point of reference) /47

$$\begin{aligned} X &= \alpha_1 (V_L - V_R) + \alpha_2 (V_F - V_R) + \alpha_3 (V_B - V_R) \\ Y &= \beta_1 (V_L - V_R) + \beta_2 (V_F - V_R) + \beta_3 (V_B - V_R) \\ Z &= \gamma_1 (V_L - V_R) + \gamma_2 (V_F - V_R) + \gamma_3 (V_B - V_R) \end{aligned} \quad (37)$$

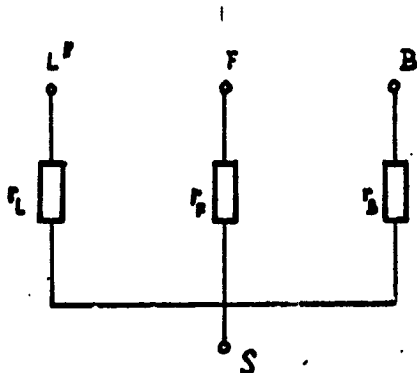


Figure 31. Addition of voltages  
having the same sign.

What we do here is make the vectorcardiograph represent voltages so that each voltage is assigned its exact "weight factor."

We shall indicate two methods by which our object can be attained.

1. The first is a method which is frequently used, but which is not all flexible. A circuit is established as sketched in figure 31, whereby in principle the above mentioned mathematical operation can be carried out. (Points L, F and B in fig. 31 represent the corresponding electrodes.) With Ohm's and Kirchoff's laws it is possible to demonstrate clearly that if resistance  $r_L$ ,  $r_F$  and  $r_B$  are large with respect to the internal resistance of system LFB, the difference of potential between point S and any other point (in our case electrode R) is the weighted average of voltages L, F and B each with that point. Thus with  $\sigma_i = 1/r_i$

$$V_S = c ( \sigma_L V_L + \sigma_F V_F + \sigma_B V_B ) \quad (38)$$

wherein the constant c is equal to  $1/(\sigma_L + \sigma_F + \sigma_B)$ . (It seems sometimes to be handier from the technical standpoint to work with conductivity  $\sigma$  instead of with resistance r.) Applying (38) the equation e.g., the X component from (33) becomes

$$X = V_S - V_R = C_X \left[ \sigma_{LX}(V_L - V_R) + \sigma_{FX}(V_F - V_R) + \sigma_{BX}(V_B - V_R) \right] \quad (39)$$

whereby, if we eliminate the constant  $c_x$  which is not very important for vectorcardiography,  $C_X$ ,  $\sigma_{LX} \sim \alpha_1$ ,  $\sigma_{FX} \sim \alpha_2$  and  $\sigma_{BX} \sim \alpha_3$ . /48

Only voltages having the same sign can be added by this method. For the general case in which there are also negative coefficients, the method must be extended. For example use can be made of a differential amplifier at one grid of which positive voltages are applied via resistors, while to the other grid, also via resistors, the potential difference is applied, provided with a negative coefficient.

Unfortunately, if coefficients  $\alpha_1 \dots \alpha_3$  are to be developed by means of resistors, a selected system of vectorcardiography cannot be simply exchanged for another. This can be understood if we bear in mind that it is true that the absolute values of components X, Y and Z in vectorcardiography do not play an important part, but that naturally all components must be accorded the same weight, which is attainable by e.g. making  $c_y$  and  $c_z$  equal to  $c_x$  (electronically signified by introduction of an extra resistor). If now one coefficient is changed--thus, one resistor--then the entire system of resistors must be adjusted to this change.

If we also realize that all resistances in the system must be large in comparison to those in the electrode system and small in respect to the input impedance of the amplifier, then we see that this rigid system is less than ideal.

2. A more flexible method is applied in the vectorcardiograph (fig. 32) as developed at Utrecht and still in use there. This instrument comprises two parts: a preamplifier and the apparatus proper, which has the three

Figure 32. The vectorcardiograph.  
(Diagram for figure 32 was omitted in original foreign text.)

cathode ray tubes. The former has five different channels (later, Chapter 11, we shall see how more than three independent leads are combined) each of which as a final stage has a cathode follower. Between each cathode follower and the succeeding differential amplifier, the voltage is divided with a potentiometer to a value that is proportional to the difference of potential between the input electrodes, multiplied by the appropriate coefficient. All voltages thus obtained from the five channels are delivered to the grids of the 49 differential amplifier via like resistors. The buttons whereby the potentiometers are operated on the righthand side of the apparatus (fig. 32). The

five columns correspond to the (maximally) five leads to be combined. The six rows correspond to the two deflecting devices of each of the three oscilloscopes. Below each potentiometer there is an additional small button whereby the sign (of the coefficient) can be selected.

The instrument is supplied with alternating current of 1 millivolt. One switch connects this signal with the various input electrodes. On each of the three oscilloscope screens there then appears a straight line whereon the correct components are given in horizontal and vertical direction by turning a knob.

The advantage of this method over the one previously described is the simple manner in which the vectorcardiograph can be adapted to one or another system of vectorcardiography. Even in the instrument here described, six different systems (or conversions, see Chapter 12) may be used (see the six setting devices at the right) which can be connected to the cathode ray tube via a selector switch. 50

Moreover because of the low output impedance of the cathode follower there is more freedom of choice with respect to the magnitude of the resistances.

Finally we will indicate a few more general features of the instrument shown in figure 32.

The three oscilloscope screens are photographed by means of a camera constructed especially for this purpose (fig. 32, left).

A so-called beam depressor is built into the apparatus to prevent the occurrence of great differences of blackening and even excess radiation in the image as the result of the varying speeds at which the dot moves across the screen. This lessens the intensity of the electron beam at low speeds at halt (heart rest interval).

Finally, the vectorcardiograph has a device to produce a punctate image,  
as indicated on page 37.

## CHAPTER 8.

### Geometrical Representation

/51

We return to the expression

$$V_P - V_{P_0} = V(\xi, \eta, t) = a(\xi, \eta)X(t) + b(\xi, \eta)Y(t) + c(\xi, \eta)Z(t) \quad (30)$$

(see page 40) which describes the relationship between the heart vector and the difference of potential between two arbitrarily selected electrodes P and P<sub>0</sub> (see page 39 and 40). Essentially this equation is a relationship between a scalar (V) and a vector ( $\overrightarrow{XYZ}$ ). A relationship of this kind can only be realized when the righthand member is the scalar product of two vectors! In other words we can also write (in simplified notation)

$$V = ( \overrightarrow{abc} \cdot \overrightarrow{XYZ} ) \quad (40)$$

in which we designate  $\overrightarrow{abc}$  the lead vector. We can also express this geometrically (fig. 33). At any instant the difference of potential between P and P<sub>0</sub> can thus be determined by multiplying the projection of  $\overrightarrow{XYZ}$  from that instant on the constant vector  $\overrightarrow{abc}$  (determined by the location of P and P<sub>0</sub>) by the absolute value of  $\overrightarrow{abc}$ , or

$$V = | \overrightarrow{abc} | \cdot | \overrightarrow{XYZ} | \cdot \cos \theta$$

where  $\theta$  is the included angle.

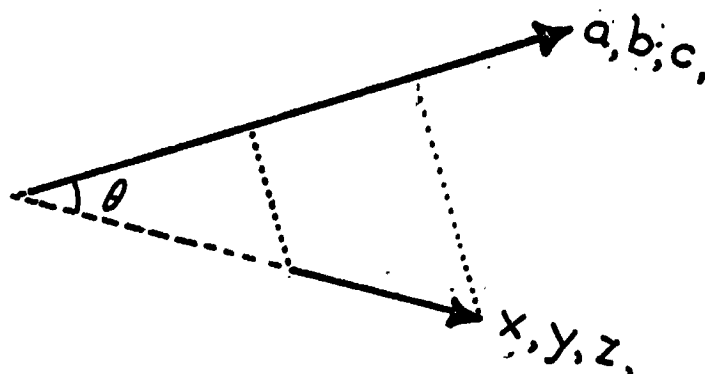


Figure 33. Lead vector.

We adapt this to the three known limb leads (right arm, left arm and left leg). We know that the sum of the three voltages is zero /52

$$V_L - V_R = a_1 X + b_1 Y + c_1 Z$$

$$V_R - V_F = a_2 X + b_2 Y + c_2 Z$$

$$V_F - V_L = a_3 X + b_3 Y + c_3 Z$$

---


$$0 = (a_1 + a_2 + a_3) X + \dots \dots \dots \quad (41)$$

If this is to be so, independent of time, then the following must be valid

$$a_1 + a_2 + a_3 = 0, \quad b_1 + b_2 + b_3 = 0, \quad c_1 + c_2 + c_3 = 0$$

or else the three lead vectors  $\overrightarrow{a_1 b_1 c_1}$ ,  $\overrightarrow{a_2 b_2 c_2}$  and  $\overrightarrow{a_3 b_3 c_3}$  must total zero when vectorally added, i.e., together they must constitute a triangle. If we now think of such a triangle with the "turning and size-changing arrow" which represents the heart vector (fig. 34) drawn inside it, then we can determine the difference of potential between the extremities at any instant by projecting the vector on the side of the triangle and multiplying each projection with the length of the matching side (the absolute value of the lead vector).

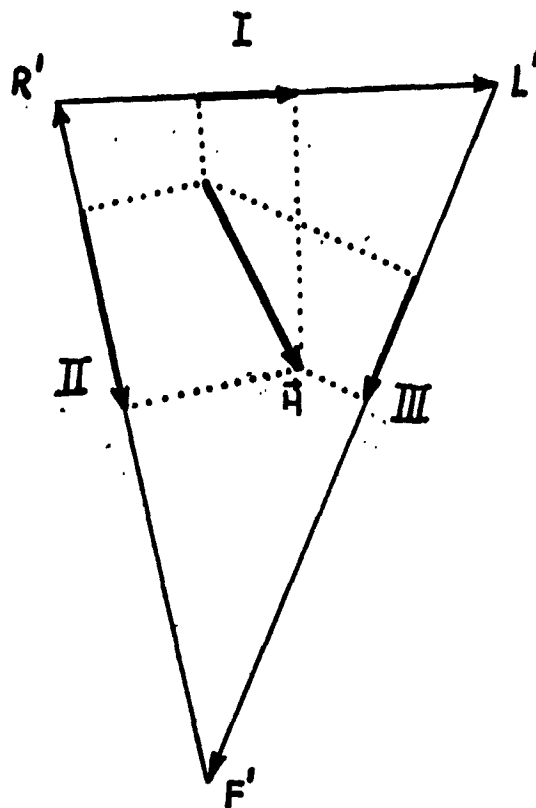


Figure 34. Lead triangle  $R'L'F'$ . The primes are used so that the apexes of the triangle will not be confused with the locations of the electrodes R, L and F on the body.

We thus arrive at a logic similar to Einthoven's and it is possible to make comparison. Just as in the equilateral triangle of Einthoven, here also the analytic formulation of the summing rule is determined by a theorem<sup>1</sup> from plane geometry. The difference is however that now the problem is 53 more firmly based and that it is no longer essential that the triangle be equilateral.

---

<sup>1</sup>If a projection is made from the sides of a triangle and if each projection is multiplied by the length of the matching side of the triangle, the sum of these products (provided with the correct signs) is zero.

It should also be noted that it decidedly is not a necessity that the vector be placed in the triangle. It will be understood that nothing is changed in the logic if we separate arrow and triangle from each other, provided that the mutual directional positioning is not changed.

Our proposition can now be generalized. We assume that we add a fourth electrode. There are then three different leads possible with R, L and F, namely each of these combined with the fourth. Each of these leads implies a lead vector, which is fixed in magnitude and direction. We consider further that at each setting of three electrodes the above mentioned triangle rule is valid: we can then form an idea of the spatial analog of the lead triangle. It must be a tetrahedron. Here also we determine the difference of potential between two points (or electrodes) at a specific instant, by multiplying the length of the connecting sides with the projection of the heart vector on the side.

The task now is not to find out what occurs with four electrodes, but to discover what happens with an expansion of this number. If we take a fifth electrode and determine its lead vector, this in our spatial representation, e.g. extending from point R, will lead to a specific point in space. This point is then correlated with the location of the fifth electrode on the human body or model. We proceed with a 6th, 7th, 8th ...  $\infty$  electrode. This develops what is designated as "image surface." In accordance with the above operation we can establish an image point on the image surface for any point on the body surface. (We must note that the construction of this image surface is entirely based on the "dipole approximation.")

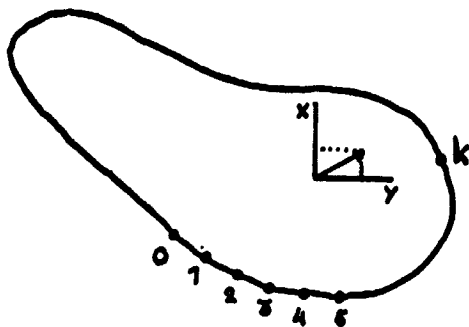


Figure 35.

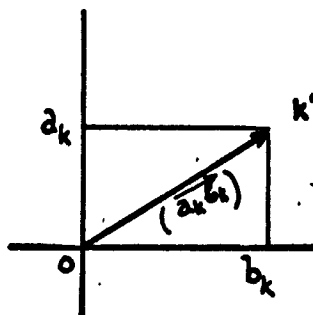


Figure 36.

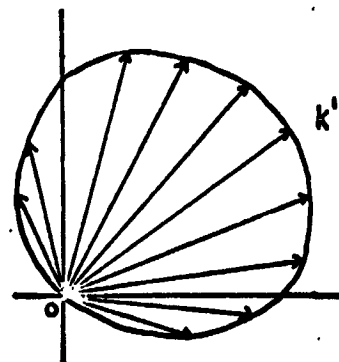


Figure 37.

Example of the construction of an image surface.

In the following discussion we shall distinguish the location of the electrode and the image thereof in the image space by providing the latter with a prime. Thus R and R' etc. (In fig. 34 we have already applied this notation)

/54

Using simple examples, we shall examine this geometric representation more closely.

1. We imagine a two-dimensional conductive layer with an arbitrary boundary (fig. 35). Somewhere in the layer we assume the presence of a known stationary dipole.<sup>1</sup> Finally, if we select at some location e.g. on the periphery (the analog of the surface of the human body) a zero point, we can measure the potentials at any point on the periphery. For such a point k

$$V_k - V_0 = a_k X + b_k Y \quad (42)$$

is valid.

<sup>1</sup>In practice the layer will not be two-dimensional, but have a finite thickness.

In order to have agreement on the result of the experiment with the two-dimensional case, the dipole is constructed as two parallel wires, placed horizontally on the layer, a so-called dipole line.

We orient the dipole so that its vector is directed along the  $X$  axis: the  $Y$  component is then zero, and

$$(V_k)_x - V_0 = a_k X. \quad (43)$$

If we then make  $X = 1$ , then (43) becomes

$$(V_k)_x - V_0 = a_k. \quad (44)$$

In an analogous manner, by making the dipole vector coincide with the  $Y$  <sup>52</sup> axis, we obtain a value for  $b_k$ . The two components of the lead vector are thus known, and we can construct  $(\vec{a}_k \vec{b}_k)$ . For this purpose we assume a system of coordinates whose origin is the representation of the zero point  $(V_0)$  <sup>(fig. 36)</sup> on the periphery of our chosen object. If we do this for "each" point  $k$  on the periphery, we obtain a fan of arrows. If the arrow points are now interconnected, the "image surface" of the conductive layer is constructed. Figure 37 shows a schema thereof.<sup>1</sup>

A point on the image surface is established in this way for any point on the actual surface. The lead vector of a pair of electrodes (here thought of as being on the periphery of the conductive surface) is obtained by connecting the corresponding points of the image surface with each other. The correlation between the points of both surfaces must be known, of course: this correlation can be given most conveniently by drawing the object within the image surface and connecting the corresponding points by lines (see fig. 39 and fig. 40). (This is possible of course only in two-dimensional cases.) It

---

<sup>1</sup>No particular value is attached to the form of this image surface (fig. 37). The choice is arbitrary, neither based on actual measurements nor upon calculation.

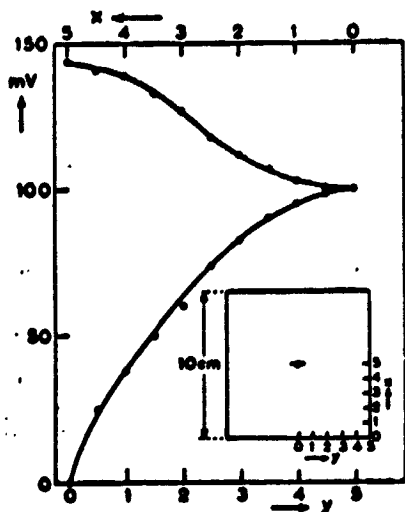


Figure 38. Measured distribution of potential.

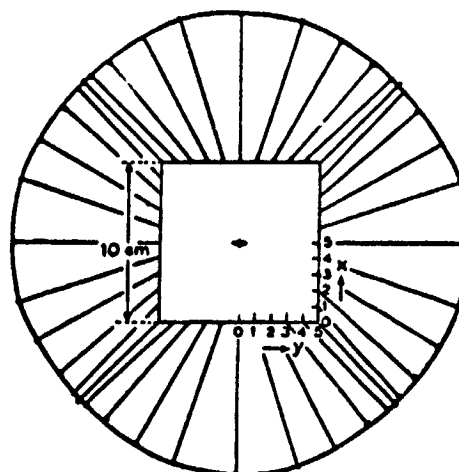


Figure 39. Square with its image surface.

must emphatically be noted that these lines have no meaning other than that which they have here for this particular purpose.

2. The following example gives the result of an experiment. A dipole (better a dipole line; see page 54) is placed in a square tank filled with electrolyte, so that the dipole vector is directed on the Y axis. The distribution of potential along the periphery of the tank with reference to an arbitrary zero can be measured. Figure 38 presents the result of these measurements for one quadrant of the square, whereby the point of reference is at the middle of one of the sides. Without going into details on the method, it can be seen that the image surface can be determined by it, in a manner similar to that described in paragraph 1. 56

(For the sake of symmetry in this case a separate measurement of a dipole oriented on the X axis can be omitted.) Figure 39 shows the image surface in which each point is correlated with a point on the square.



sphere surface:  $dV/d = 0$ . This necessitates the addition of an extra term, which furthermore like the above equation must satisfy Laplace's law:  $\nabla^2 V = 0$ . By testing, we determine that this term must be  $K \cdot r \cos \theta$  wherein  $K$  is a constant which is independent of  $r$  and  $\theta$ , which must be so adapted that it satisfied the limit condition. (We can show that  $K \cdot r \cos \theta$  is the only 57 form that satisfies it.) If we write

$$V = \frac{D_v \cos \theta}{4 \pi r^2} + K r \cos \theta \quad (45)$$

by differentiation of  $r$  in  $r = a$  we can determine the constant  $K$

$$K = \frac{2 D_v}{4 \pi a^3} \quad (46)$$

The distribution of potential on the sphere surface ( $r = a$ ) now becomes

$$V = \frac{3 D_v \cos \theta}{4 \pi a^2} \quad (47)$$

In the interests of symmetry, and also from (47) it follows that the image surface is a sphere.

4. We assume finally a conductive closed cylinder with the dipole on the long axis, above the middle (a crude approximation of the human back). Although with much more difficulty than in example 1, everything here can also be determined. (We do not give the calculation)

Figure 40 shows the cylinder with its image surface. The corresponding points are connected by lines, of which as we have said before, 58 direction and length have no significance.

The lead vector for any pair of electrode locations on the actual body can now be directly determined, both in direction and magnitude, by converting the corresponding points on the image surface.

In figures 39 and 40 we can correlate a few general concepts.

a. The discontinuities in the direction of the periphery of the body disappear in the image surface, while sharp edges are smoothed in the image. The image surface itself is nearly spherical. A general demonstration of this for the three-dimensional case has not yet been given. It well may be for the two-dimensional case that the image "surface" of a body with any given boundary is a circle. The proof has not yet been published.

b. If we add "extremities" to the cylinder, they should only be expressed in the image surface as small elevations. This can be understood if we recall that the electric current field penetrates only slightly into the extremities and that thus the difference of potential between two points thereon is small. This must then be expressed in small lead vectors.

c. The question can be asked, must we limit ourselves to the image surface? That is, have points inside or outside this surface physical significance? The answer is affirmative and it is also meaningful to speak of an image space. We shall explain this.

1. Given that we arrange an electrode inside the body (a method often used today) and that we measure the difference of potential between this and an electrode on the outside, the result will be greater, the nearer <sup>or</sup> the internal electrode is to the heart (i.e., the electric source). This appears directly from a consideration of the formula for the potential developed by a dipole. In the equation (see page 40)

$$V = aX + bY + cZ.$$

(30)

this larger value for a, b and c is implied as the electrode approaches /59 the heart. Translated into terms of the image space, this will mean that the image point of the internal electrode falls outside the image surface (a strong indication of this is lacking). Without going into the background, let it be noted that the image of the heart itself--conceived as a point (dipole)--must be thought of as in infinity.

2. It can be seen as a consequence that also points in the image space inside the image surface have significance. Assume two electrodes on the body, e.g. R and L. We connect these via two large resistors  $r_1$  and  $r_2$  with a point S (fig. 41a). Then the difference in potential between any point P and S will be

$$V_S - V_P = \frac{r_1}{r_1 + r_2} (V_L - V_P) + \frac{r_2}{r_1 + r_2} (V_R - V_P). \quad (48)$$

If we take  $V_L - V_P = V_1$ ,  $V_R - V_P = V_2$  and  $r_1 + r_2 = r_t$ , then

$$V_S - V_P = \frac{r_1}{r_t} \cdot V_1 + \frac{r_2}{r_t} \cdot V_2. \quad (49)$$

Applying (30) we can write this as

$$V_S - V_P = \left( \frac{a_1 r_1 + a_2 r_2}{r_t} \right) X + \left( \frac{b_1 r_1 + b_2 r_2}{r_t} \right) Y + \left( \frac{c_1 r_1 + c_2 r_2}{r_t} \right) Z \quad (50)$$

or

$$V_S - V_P = aX + bY + cZ \quad (51)$$

in which a, b and c are the components of the lead vector of P at S. It /60 will be readily perceived that S', the image of S, lies on the line connecting

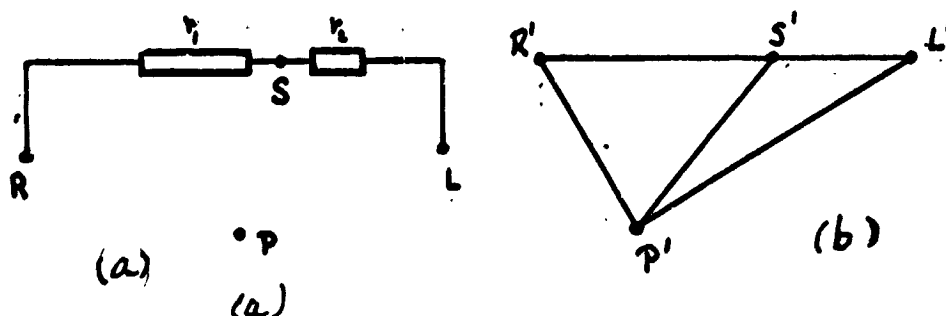


Figure 41. (a) By connecting R and L via two resistors  
 (b) we develop point S.  $S'$ , the image of S, is on line  
 $R'L'$ . Thus  $R'S':S'L' = r_1:r_2$ .

$R'$  and  $L'$ , said line being divided into segments with the relationship  $r_1:r_2$  (fig. 41b).  $S'$  is inside the image surface.

In general any point can thus be reached inside the image surface by arranging a potentiometer between two electrodes P and Q. The image of the sliding contact is moved by turning the knob, along the connecting line  $P'-Q'$ .

## CHAPTER 9.

### Application of the Geometric Representation

In this chapter we shall consider, with reference to a number of 61 examples, how the concept of image space that we have described can be applied. As preface it must be stated that we do not make the claim that every application is actually meaningful or even that the individual method is always suitable for achieving a specific purpose.<sup>1</sup> On the contrary a purely analytic approach often performs the required task more exactly and sometimes more rapidly besides. We emphasize the point that we also put analysis in first place: that the image surface can nonetheless support the determination of a clear concept is demonstrated by what follows:

1. We return to the Central Terminal (C.T.) introduced by Wilson (see page 19). The three vertexes R', L' and F' which together constitute an oblique triangle (see page 52) are the image points of the electrode positions on the right arm (R), Left arm (L) and left leg (F). We observed that the sides of the triangle have become significant through introduction of the concept of the lead vector (page 51).

It is now easy to see (see page 59 and (4), page 20) that the image of the C.T. is determined at the center of gravity of the triangle (see fig. 42a). Naturally also the foot points of the line of gravity can be physically realized. In addition the extremities are connected by large and like resistances to points A, B and C (fig. 42b). These are determined by transformation back

---

<sup>1</sup>Naturally the image space is a geometric representation of analytic data. It never teaches more than does the basis on which it stands.

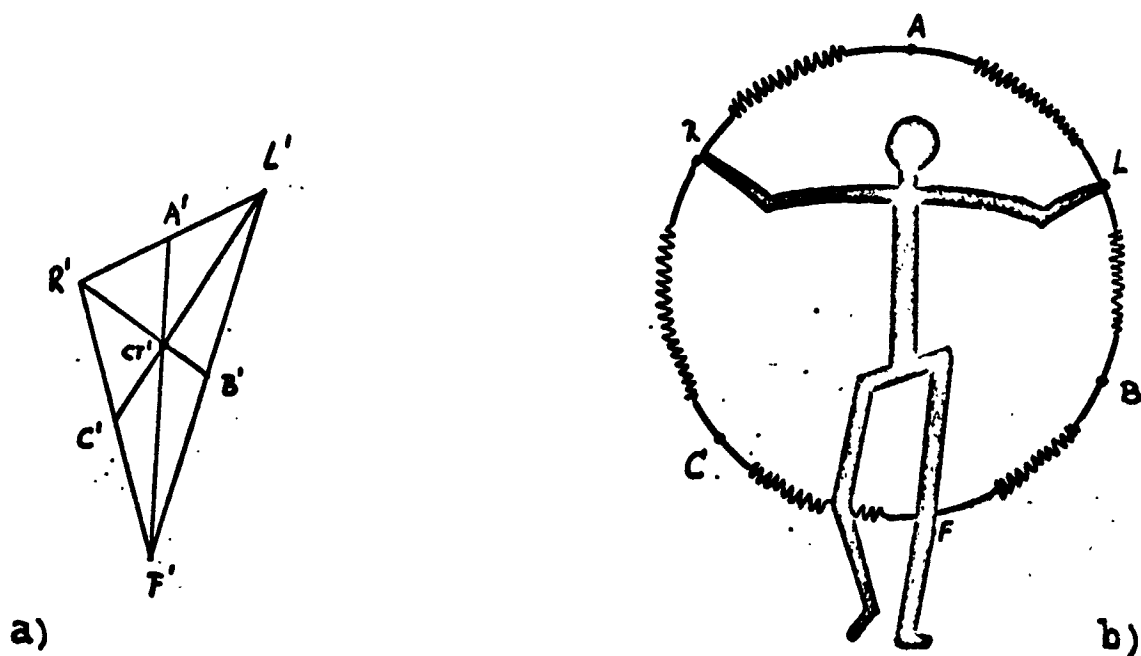


Figure 42. The foot points of the lines of gravity of the lead triangle (a) can be realized physically (b).

into the image space on the middle of the sides of the triangle ( $A'$ ,  $B'$ ,  $C'$ ) (see fig. 42). (In practice this "distribution" in the apparatus takes place after one or more amplifier stages).

/62

Instead of the many used leads  $R-CT$ ,  $L-CT$  and  $F-CT$ , the differences of potential  $R-B$ ,  $L-C$  and  $F-A$  can now be recorded. Thus results are obtained which are 1.5 times larger than the originals (justifiably designated "augmented leads"). This follows geometrically from the fact that the gravity point of a triangle is met on  $1/3$  of each of the lines of gravity. Algebraically the problem is somewhat less complex and also it is direct.

How far the following argument is useful, we leave to the reader's judgment.

2. It will be methodologically advantageous that we will be able to place three pairs of electrodes in such a way that their differences of potential exclusively will be caused by the  $x$ ,  $y$  and  $z$  components of the heart vector, so that

$$X = \alpha_1 V_1, \quad Y = \beta_2 V_2, \quad Z = \gamma_3 V_3 \quad (52)$$

(see (33) page 40).

The internal and external structure of the human body does not permit us to abandon natural geometry. But if we know the image surface of the body (and the relationship that connects each point thereof with a point in the actual body) then we can seek two points thereon, of which the connecting line (naturally, the lead vector) runs parallel to e.g. the X axis of our chosen system of coordinates. If we determine by transformation of these two points to the corresponding locations on the human body, we then have found a pair of electrodes which satisfies the requirement for the X component. /63

For each of the components,  $\infty^2$  pairs of points can thus be determined. The magnitude of the difference of potential which can be measured between them is dependent upon the length of the corresponding lead vector and thus it can be arbitrarily selected within certain limits. The method sketched here suggests a system of six electrodes. It will be readily seen that the task can be accomplished with four also, in which case one is common. The [ four electrodes can be so selected that the three lead vectors are of equal length (with only two degrees of freedom). That is to say, the factors from (52) are equal

$$\alpha_1 = \beta_2 = \gamma_3.$$

The practical usefulness of the matter here discussed is very limited, unfortunately, because a detailed investigation requires determination of the relationship between image surface and human body.

3. In conclusion we will observe that the search for the long connecting line in the image space will lead to determination of the electrode location that yields the big difference in potential.

In general we can say that the concept of image space can help us in finding a physically well founded system of vectorcardiography.

Under a system of vectorcardiography we understand a specific choice of electrode locations and a rule for the production of the measured voltages (form of linear combinations) so that therefrom three time functions are determined which--each independently--represent the three components of the heart vector. If all investigators could agree on such a system, all other systems now in use which are often based on intuition could be discarded in favor of it. With this the uniformity so devoutly wished for and so necessary would be brought into vectorcardiographic research which is so important and which is being applied in so many parts of the world. However obvious this may seem, we can form an idea of what all this really signifies by reflecting that an enormous store of information has been collected in the course of many years--with intuitive systems. Many cardiologists establish their diagnoses on the basis of their own observations with systems /64 thus chosen. It will therefore certainly not be easy to realize the indicated ideal. What steps could be undertaken to draw together the conflicting viewpoints will be discussed elsewhere (Chapter 12).

If a proposal is presented, then it is naturally important that the principle representation of the electric activity of the heart by a vector--i.e., the dipole approximation (page /38)--no longer be the subject of doubt.

We shall now consider what methods there are to test the accuracy of the dipole approximation, and the conclusions these tests have lead to.

## CHAPTER 10.

### Testing the Dipole Approximation

#### 1. Cancellation method

Following the historic development, let us first take a method based /65 on the geometric representation.

We assume the image surface of the human body to be approximately spherical (fig. 43) (For the sake of clarity an exact sphere is drawn in figure 43. No use is made of the properties of an exact sphere in the discussion. Somewhere on the said surface there lie points  $R'$ ,  $L'$  and  $F'$ : inside the sphere, at the center of gravity of triangle  $R'L'F'$ , we find the CT image (Chapter 9, page /61). If we select an arbitrary point  $P$  on the sphere surface, we can then record the voltage characteristic as function of time between said point and the CT. (In fact we mean naturally the difference of

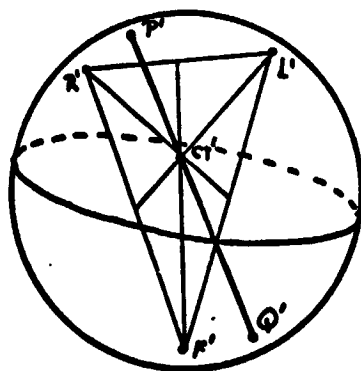


Figure 43. The image of the human body represented as a sphere with images of  $R$ ,  $L$ ,  $F$  and  $CT$  and of two electrodes  $P$  and  $Q$ , which are antipodal to each other, inside it.

potential between the electrodes placed at corresponding points on the body, determined by transformation.) Since hardly any confusion can arise concerning it, for the sake of the simplicity of our narrative we shall omit this addition in the following pages.

P will be positive with respect to CT when the projection of the heart vector on line P'CT' at P'"points" negative, in case this projection points near CT'

We now assume that since the dipole (vector) approximation is accurate, point Q to be determined must be such that the difference of potential between Q and CT as a function of time produces a proportional curve of opposite sign (fig. 44). This point will of course be the point of intersection of the 66 connecting line P'CT' with the other edge of the sphere (fig. 45). The projection of the heart vector ( $\vec{H}$ ) on Q'CT' will have the same value as the projection of P'CT but it will have the opposite sign.

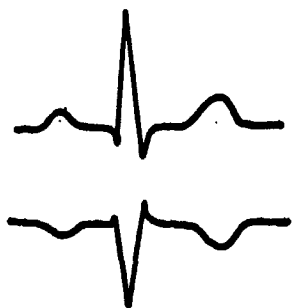


Figure 44. Two proportional electrocardiograms of opposite sign.

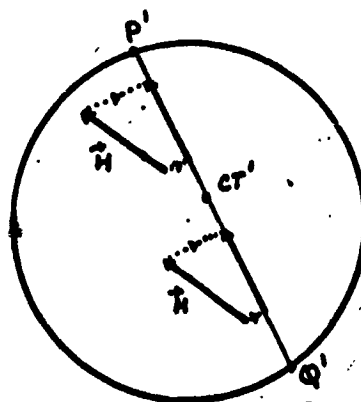


Figure 45. The projections of  $\vec{H}$  on the lead vectors P'CT' and Q'CT' are equal but the lead vectors are unequal in length.

We can test this in practice by the two leads  $V_P - V_{CT}$  and  $V_Q - V_{CT}$ , furnished with a proportioning factor (a potentiometer), by connecting them, in opposition to each other, to the same pair of plates of an oscilloscope. If a location P is selected anywhere on the body it is possible to try to move electrode Q until a point is found at which the dot on the screen becomes stationary: the two electrocardiograms exactly balance each other ("cancellation").

Control can always be applied in this test by means of the potentiometer, to make the result minimal.  $V_Q - V_{CT}$  will only be proportional with respect to  $V_P - V_{CT}$  and not alike in absolute values, since the lead vectors are generally not alike in length (see (40) page 51).

That point Q is experimentally sought is a consequence of the fact that in actuality the image surface of the test individual is not known and thus the true surface cannot in reality be determined from it by a transform operation.

The above described method has a shortcoming, namely that as long as 67 the dot moves on the screen it cannot be seen directly whether this is because the exact point of Q has not been found or whether it is only because the potentiometer is poorly adjusted. Experimentally it is therefore more con-

venient to use both pairs of oscilloscope plates, connecting a lead to each of them. As long as the exact location of Q has not been determined, a loop appears on the screen that "snaps into" a line where the point sought for is found. Using the potentiometer, it is possible to turn this line by  $45^\circ$ , whereby the conditions for the halting of the dot in the first case are established.

A practical difficulty is that in the displacement of an electrode the measuring apparatus is subjected to such strong noise signals that the instrument needs some time to recover from this "shock" even when a high speed starter is built into it. The phenomenon makes the experiment cumbersome and time-consuming, in contrast to the method described below (Chapter 10, 2, page 68).

In general, satisfactory results are obtained by the described experiments. It appears to be possible in a great many cases, with a given point P, to find its antipode Q (otherwise this phenomenon does not reveal very much. See comment, page 74).

However it appears now and then that even with approximation one EKG cannot be made successfully to compensate another. The failure of the cancellation is also occasionally caused by the performance of completely independent experiments. Without going deeply into this we will say a few words about it.

The failure of the cancellation sometimes coincides with a strikingly poor agreement of different systems of vectorcardiography (see Chapter 12). In seeking a clarification of these phenomena we have the choice of two possibilities.

a. The heart itself is the cause. E.g., if the measured values are so great that higher polarities (see Chapter 15) play a relatively important role.

b. There is something particular about the conductivity of the tissues outside the heart. To gain more information on this, it is necessary to have available material on heart beats of which the electrical phenomena are completely different from the normal. Peculiarities in conductivity outside the heart should then be expressed for these heart beats determined on the outside in the same way as normal beats. 68

7

Nature itself bids us experiment with possibilities in this direction, since such outside beats are found in the so-called extrasystoles. For extrasystoles it is true that excitation of the heart is not in the majority of cases transmitted by the sinus node, but the heart beat begins anywhere in the myocardium to spread in a completely abnormal manner over the heart, thereby giving rise to a vectorcardiogram that differs drastically from the normal.

Results of experiments where these extrasystoles are found appear to have shown that the difficulties must be sought not outside but in the heart or on its surface.

Returning to the cancellation, the following should be noted. If in the cases in which compensation is satisfactory, electrode P at the identical location is chosen in various test persons, point Q will by no means be in the identical location. Clearly, individual differences play a great role. This means that the relationship between the electrocardiogram that can be taken on the exterior of a human body and the actual electrical phenomena in the heart vary from person to person. That essential uncertainty arises for electrocardiography because of this is obvious. We will return to these problems later (Chapter 12, page 93).

## 2. Becking's method of cancellation

Earlier (page 40) we saw how three arbitrary independent leads can give us the components of the heart vector (see (33)). If we add a fourth lead to these three (possibly by means of a whole new set of two electrodes) we can, in analogy to (32) page 40 write four equations

$$\begin{aligned}
V_1 &= a_1X + b_1Y + c_1Z \\
V_2 &= a_2X + b_2Y + c_2Z \\
V_3 &= a_3X + b_3Y + c_3Z \\
V_4 &= a_4X + b_4Y + c_4Z
\end{aligned}
\tag{53}$$

Since our assumption about the heart vector (that is, the dipole /69 approximation) is accurate, these four equations are dependent. We can then eliminate X, Y and Z and then the following must be valid.

$$\begin{vmatrix} V_1 & a_1 & b_1 & c_1 \\ V_2 & a_2 & b_2 & c_2 \\ V_3 & a_3 & b_3 & c_3 \\ V_4 & a_4 & b_4 & c_4 \end{vmatrix} = 0.
\tag{54}$$

(Becking's equation)

This implies that each of the four differences of potential, independent of time, can be represented by a linear combination of the three others.

$$V_4 = p_1V_1 + p_2V_2 + p_3V_3 \tag{55}$$

( $p_1$  is the subdeterminant of  $V_1$  divided by the subdeterminant of  $V_4$ ).

With this we have achieved the possibility of performing experiments similar to those described previously,--a kind of generalized cancellation. For example let  $V_4$  cause the vertical deflection of the oscilloscope screen dot. A linear combination of  $V_1$ ,  $V_2$  and  $V_3$  ( to be effected by a balancing amplifier, potentiometers and switches, in order to be able to adjust the magnitude and sign of factors p)--will be connected on the other pair of

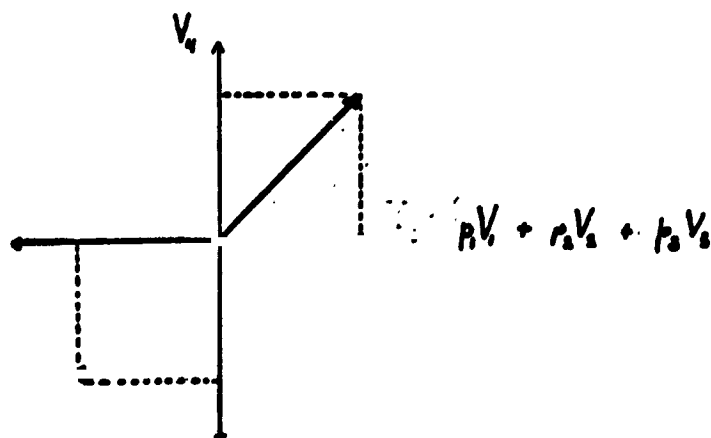


Figure 46. Becking's method of cancellation.

plates, and it must again be possible to let the dot describe a closed line at a  $45^\circ$  angle (fig. 46).

An advantage of this method over the other--as already indicated (page 67)--is that here no electrodes need be displaced, so that 70 the measuring apparatus does not get out of order.

The results are rational as in the above discussed cancellation method. That is to say, within a variation of a few percent the goal can be realized. But here also we encounter cases in which the cancellation even in approximation will not succeed. As in the first case, we must come to the conclusion that within fairly wide limits vector representation in general yields a good description of the phenomena that we are investigating, but that a more exact representation is needed.

The Becking equation affords not only the possibility of testing dipole approximation. It also puts us in the position to determine the image surface of a test person. We shall discuss this in more detail. If we substitute equation (53) for  $V_1, V_2, V_3, V_4$  in (55) we see immediately that

$$\begin{aligned}
p_1 a_1 + p_2 a_2 + p_3 a_3 &= a_4 \\
p_1 b_1 + p_2 b_2 + p_3 b_3 &= b_4 \\
p_1 c_1 + p_2 c_2 + p_3 c_3 &= c_4
\end{aligned}
\tag{56}$$

must be valid. If in other words, e.g. by model measurements, the coordinates (with reference to an arbitrary zero point) of electrodes 1, 2 and 3 are determined in the image space, then it is possible to go farther and determine the location on the image surface of any electrode location on the living brain (naturally valid only if the dipole approximation is valid).

It can be formulated in still another way. We write (56) in geometric form

$$(\overrightarrow{a_4 b_4 c_4}) = p_1 (\overrightarrow{a_1 b_1 c_1}) + p_2 (\overrightarrow{a_2 b_2 c_2}) + p_3 (\overrightarrow{a_3 b_3 c_3}) \tag{57}$$

Factors  $p_i$  are scalars.

If we assume for the sake of convenience that the four lead vectors  $(\overrightarrow{a_i b_i c_i})$  ( $i = 1, 2, 3, 4$ ) have the same origin (this means that the four leads  $V_i(t)$  have a common electrode) then the lead vector  $(\overrightarrow{a_4 b_4 c_4})$  and hence the location of the fourth unshared electrode is determined in the image space by geometrical means (fig. 47) after  $p_1$ ,  $p_2$  and  $p_3$  have been experimentally determined. /71

One final comment which is analogous to that in Chapter 10, 1: with (56) and (57) we have gained a test of the practical utility of vector representation. For different test individuals we must find like factors  $p$  by identical placing of electrodes 1, 2, 3 and 4 and thus the image of electrode 4 must always be found in the same place. Otherwise expressed, the image surface must have

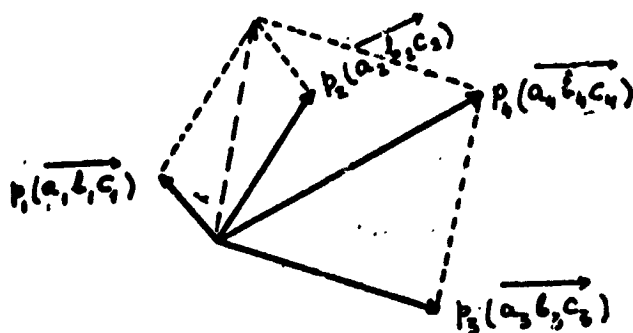


Figure 47. Geometric (image spatial)  
representation of (57).

the same form and measurements for all subjects. Unfortunately the results of these experiments leave ~~much~~ to be wished for, in consequence of individual differences. However the range of difference is such that physically based and practically usable systems of vectorcardiography can be found.

### 3. Transformation method

Assuming that we have two systems of vectorcardiography (see Chapter 9, 3, page 63) K and L, such that only three leads play a role in each case, and they are accurate enough in principle for determination of the other components of the heart vector, these two systems can be defined by three equations each (see (33) page 40)

$$\begin{aligned} X_K &= \alpha_{1K} V_{1K} + \alpha_{2K} V_{2K} + \alpha_{3K} V_{3K} \\ Y_K &= \beta_{1K} V_{1K} + \beta_{2K} V_{2K} + \beta_{3K} V_{3K} \\ Z_K &= \gamma_{1K} V_{1K} + \gamma_{2K} V_{2K} + \gamma_{3K} V_{3K} \end{aligned} \quad (58)$$

and

$$\begin{aligned} X_L &= \alpha_{1L} V_{1L} + \alpha_{2L} V_{2L} + \alpha_{3L} V_{3L} \\ Y_L &= \beta_{1L} V_{1L} + \beta_{2L} V_{2L} + \beta_{3L} V_{3L} \\ Z_L &= \gamma_{1L} V_{1L} + \gamma_{2L} V_{2L} + \gamma_{3L} V_{3L} \end{aligned} \quad (59)$$

If we consider that neither  $\overrightarrow{(X_K, Y_K, Z_K)}$  nor  $\overrightarrow{(X_L, Y_L, Z_L)}$  is the actual [72]  
heart vector but only an approximation, as close as the developer of the system  
in question could make it, and if we realize also that each of the differences  
of potential  $V_{1K} \dots V_{3L}$  is a linear function of the true components,  $X, Y, Z$   
of the heart vector, we can see that  $X_K, Y_K$  and  $Z_K$  can be written as  
linear functions of  $X_L, Y_L$  and  $Z_L$ . If  $V_{1K} \dots V_{3L}$  are linear functions of  
 $X, Y$  and  $Z$ , we can introduce these functions in (58) and (59) with elimination  
of  $X, Y$  and  $Z$ . Thus

$$\begin{aligned} X_K &= p_x X_L + q_x Y_L + r_x Z_L \\ Y_K &= p_y X_L + q_y Y_L + r_y Z_L \\ Z_K &= p_z X_L + q_z Y_L + r_z Z_L \end{aligned} \quad (60)$$

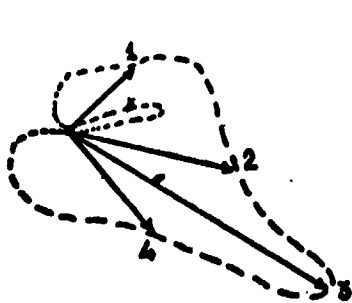
It is obvious that the above statement is only accurate because the dipole  
approximation is valid. In other words, the testing of the validity of a  
linear transformation of the form of (60) (actually (60) is none other  
than a transformation formula) is equivalent to the testing of the dipole  
approximation.

Anticipating a problem, which we shall discuss in more detail later, we  
note here that since systems K and L should both yield the same results, it  
is valid to say

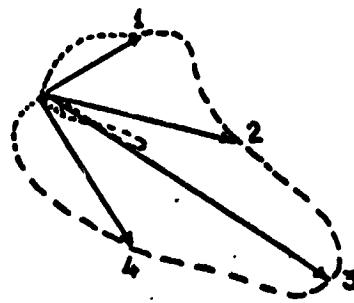
$$X_K = X_L \quad Y_K = Y_L \quad Z_K = Z_L \quad (61)$$

This means that transformation (60) is an identical transformation,  
written as

$$\begin{pmatrix} p_x & \dots & r_x \\ p_y & \dots & r_y \end{pmatrix} \begin{matrix} 1 \rightarrow 1 \\ 2 \rightarrow 2 \\ 3 \rightarrow 3 \end{matrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (62)$$



System K



System L

Figure 48. Two VKGs, representatives of two systems of vectorcardiography. The arrows indicate the heart vector in isophasal position.

This actually holds in that when the matrix elements outside the 73 main diagonal are small and the diagonal elements are approximately one, the systems presumably correspond. Both would then in general acceptably approximate the behavior of the heart vector at the time of the heart beat.

Naturally it is possible to approximate experimentally the essence of the linear transformation (60) by comparing two vectorcardiograms--representatives of systems K and L--taken from the same heart under identical circumstances.

We shall elucidate this with reference to figure 48 wherein two "loops" (which must here be thought of as three-dimensional) are drawn. By introduction of time markers into the recording we can seek synchronous (isophasal) points and thereby measure the approximate coordinates  $X_{1K}, Y_{1K} \dots Z_{1L}$ . For

each pair of points K, L equation (60) must be satisfied. The nine coefficients  $P_x \dots r_z$  are not known, however. We can find them by considering three pairs of points ( $i = 1, 2, 3$ ) wherein  $3 \times 3 = 9$  equations can be set up so that they can be solved for the nine coefficients. If we now select an arbitrary fourth pair of points and substitute their coordinates in the system of three equations according to (60) with the coefficients calculated for them, we will encounter no contradictions, on condition ... that the dipole approximation is valid!

Naturally the values of  $p_x \dots r_x$  are not completely independent from the selection of points  $i$ . In practice preferably more than three 74 are selected so that for the said nine unknowns more equations exist. Using the method of least squares--and an electric calculator--we obtain values that as closely as possible represent the case in question.

We will make one more comment before we come to our conclusion. We consider two entirely independent "loops," not random, but apart from that arbitrarily selected. A linear relationship can almost always be found in that one figure can be transformed with reasonable accuracy to the other (this is not true of detailed figures). It can happen that the finding of a transformation that tallies is no guarantee of the validity of the dipole approximation. This in contrast to the transformation that does not tally--as may primarily be the case with pathological loops--in which the approximation must naturally be rejected directly. We must therefore handle our results meticulously.

Our conclusion, in analogy to the earlier one is this: the dipole approximation for every test individual certainly does not in itself

deserve to be discarded and it can doubtless serve as a reasonable provisional working base, but the individual differences are of such a nature that an adaptation and extension of theory and method deserve full consideration (see Chapter 15).

## CHAPTER 11.

### The Total Dipole and Corrected Systems

Up to this point in our discussion we have always proceeded from the dipole approximation, i.e.: the electric heart activity can be represented by a vector, the dipole moment by one dipole which is stationary. In the preceding chapters we have seen that this representation can lead in general to satisfactory results, but that sometimes the approximation falls short of the mark. It is therefore reasonable to extend our hypothesis. We do this by assuming a collection of dipoles and by investigating to see to what extent the distribution of potential on the body surface yields information on distribution of dipole density in the body.

This idea can be thrown out immediately in such a formulation because fundamentally it is not possible to determine the dipole distribution in a body except by measurement of the potential at the surface (A conclusion already reached by Gauss with reference to the matter of the origin of terrestrial magnetism.)

The uncertainty of this problem will be demonstrated by a simple example: we imagine a closed surface with a homogeneous distribution of dipoles thereon. There is then a dipole moment perpendicular to each surface element, proportional in size to the surface of the element (fig. 49). It will be recognized that such a layer of dipoles presents a discontinuity in the potential at the layer. This potential jump will occur at the surface itself. As a result of it an arbitrary distribution of potential somewhere

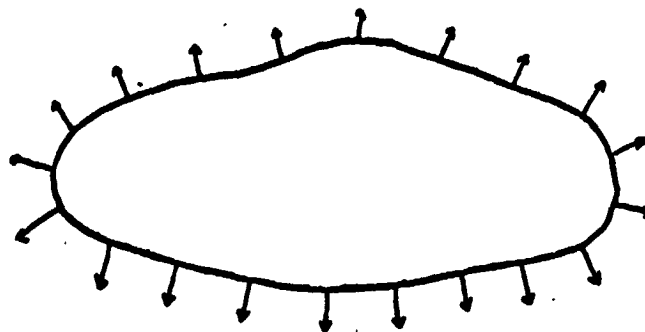


Figure 49. Homogeneous, closed dipole layer.

outside this closed surface, will not be influenced by the presence of this surface, no matter what the configuration thereof. It is relevant to our problem that if one or more homogeneous closed dipole layers are to be added to a specific dipole distribution in the heart, the potential distribution on the body surface must not be altered. Conversely, a given potential distribution on the body surface yields no decisive information on the internal dipole distribution. What can be determined from the potential distribution, however, is the total dipole effect, hence the total dipole moment, of any dipole distribution within a body, since the body is a homogeneous and isotropic medium. The theoretical solution of the problem a few years ago (1954) by Gabor and Nelson was--like the result--astonishingly simple.

A space (fig. 50) is assumed, insulated to the outside and consisting, as already noted, of a homogeneous and isotropic medium. In the space the authors posit a collection of positive and negative current poles, so that the current field  $\vec{j}$  in the body comprises as a totality neither positive nor negative sources, hence:

$$\int_{vol} \text{div } \vec{j} \, dv = 0 \quad (63)$$

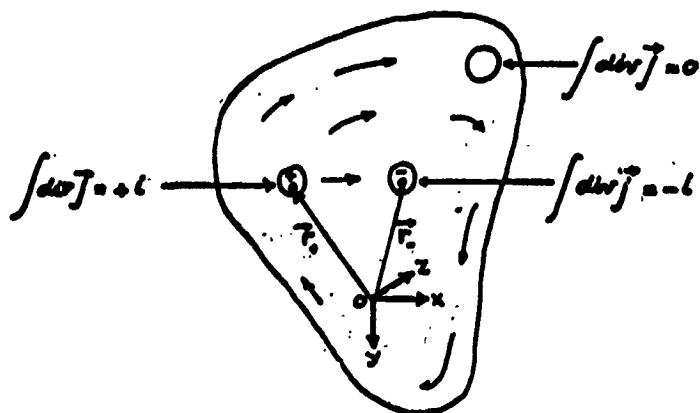


Figure 50. Positive and negative current poles in an insulated homogeneous and isotropic medium.

We can formulate the insulated condition of the body by noting that the normal component of the current density at the boundary is zero. In other words, /77

$$\vec{j} \perp \vec{dS} \quad (64)$$

We now consider one plus and one minus pole which deliver a current passing into and out of the body. The radius vectors at these poles from the source of an arbitrarily assumed system of coordinates are designated  $\vec{r}_+$  and  $\vec{r}_-$ . The product of current intensity  $i$  and the vector  $(\vec{r}_+ - \vec{r}_-)$  is designated  $\vec{D}_1$ , the "doublet intensity."

$$\vec{D}_1 = i(\vec{r}_+ - \vec{r}_-) \quad (65)$$

(as we indicated earlier (page 26) within limits  $(\vec{r}_+ - \vec{r}_-) \rightarrow 0$  and  $i \rightarrow \infty$ ,  $\vec{D}_1$  is the dipole moment). We shall see how this expression (65) can be generalized.

We begin with a step which is perhaps not entirely justifiable mathematically,

but which nonetheless leads to a satisfactory result. We imagine each pole to be "smeared out" over a volume element  $\Delta v$ . It will be immediately seen that the divergence of the current density, integrated over the volume element, yields the current  $i$  that passes at the pole. Thus, for a positive pole

$$\int_{\Delta v} \text{div } \vec{j} dv = i \quad (66a)$$

(wherein  $\Delta v \gg dv$ ) and, in analogy, for a negative pole

$$\int_{\Delta v} \text{div } \vec{j} dv = -i \quad (66b)$$

This is all valid for any current doublet in the body. Naturally everywhere where there are no poles

$$\int_{\Delta v} \text{div } \vec{j} dv = 0 \quad (66c)$$

Without going into the problems of singularities, we assume that (66a) and (66b) will be valid when we proceed from current doublets to current dipoles. It is then clear that for the total dipole moment, caused by an arbitrary collection of current dipoles, we can write

$$\vec{D}_1 = \int_{\text{vol}} \vec{r} \text{div } \vec{j} dv \quad (67)$$

We shall demonstrate that this expression can be associated with the potential distribution, as we are able to measure it on the body surface.

We first consider the x

component of (67)

[78]

$$(\vec{D}_1)_x = \int_{vol} x \operatorname{div} \vec{j} dv \quad (68)$$

According to a rule of vector calculation

$$\operatorname{div} (x\vec{j}) = x \operatorname{div} \vec{j} + \vec{j} \cdot \overrightarrow{\operatorname{grad} x} \quad (69)$$

so that

$$(\vec{D}_1)_x = \int_{vol} \operatorname{div} (x\vec{j}) dv - \int_{vol} \vec{j} \cdot \overrightarrow{\operatorname{grad} x} dv \quad (70)$$

According to Gauss' theorem we can write the first term as

$$\int_{vol} \operatorname{div} (x\vec{j}) dv = \int_{opp} x \vec{j} \cdot d\vec{S} = 0 \quad (71)$$

for  $\vec{j} \perp d\vec{S}$  (see (64)). Thus

$$(\vec{D}_1)_x = - \int_{vol} \vec{j} \cdot \overrightarrow{\operatorname{grad} x} dv = - \int_{vol} j_x dv \quad (72)$$

We saw earlier that  $\vec{j} = -\alpha \vec{\nabla} V$  (equation (17) page [25]) whereby we get

$$(\vec{D}_1)_x = \alpha \iiint \frac{\partial V}{\partial x} dx dy dz \quad (73)$$

Integrating according to x gives

$$(\vec{D}_1)_x = - \alpha \iint V dy dz \quad (74)$$

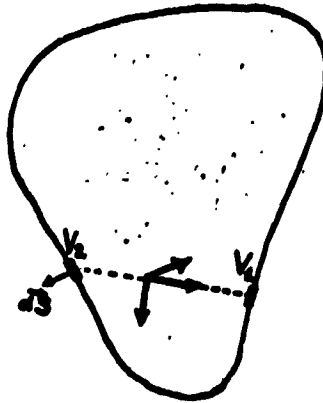


Figure 51.  $V$  (see (74)) =  $V_1 - V_2$ .

wherein  $V = V_1 - V_2$  (see fig. 51) ( $V_2$  in the negative  $x$  direction, is here assumed to be negative)  $dy dz$  is the  $x$  component of surface element  $\vec{dS}$ . Thus

$$(\vec{D}_1)_x = \sigma_{\text{opp}} \int V (\vec{dS})_x \quad (75a)$$

Analogous treatment of  $y$  and  $z$  components of  $\vec{D}_1$  yields

$$(\vec{D}_1)_y = \sigma_{\text{opp}} \int V (\vec{dS})_y \quad (75b)$$

$$(\vec{D}_1)_z = \sigma_{\text{opp}} \int V (\vec{dS})_z \quad (75c)$$

79

If we now combine the components, we obtain

$$\vec{D}_1 = \sigma_{\text{opp}} \int V \vec{dS} \quad (76)$$

This expression is indeed astonishingly simple. To obtain the total dipole it is merely necessary to integrate the potential<sup>1</sup> over the entire body surface, whereby the question of the point of application of the heart vector--thus the fixedness of the dipole<sup>1</sup> is eliminated.

Before we go into the practical significance of (76), because this equation has had such an important effect on the further development of vector-cardiography we will demonstrate the result in yet another fashion. Starting from an expression that was used earlier

$$\vec{J} = \sigma (-\nabla V + \vec{E}^*) \quad (15)$$

(see pages 25 and 28) where  $\vec{E}^*$  is the so-called motor field strength, the origin of the electric activity of the heart.

We consider as before a homogeneous conductive and infinite space. In a spherical volume separated from this space we imagine a homogeneous motor field strength  $\vec{E}^*$  caused by diffusion and membrane effect. As deduced in Chapter 5, for the potential at a point anywhere outside the sphere, we can write (in pole coordinates  $r$  and  $\theta$ ) (see page 29). 80

$$V = \frac{E^* \cos \theta \cdot \text{volume bol}}{4 \pi r^2} \quad (27)$$

---

<sup>1</sup>It seems contradictory to what has been said earlier, to speak here of "the potential" and not potential differences. It is not a contradiction however because nothing is changed when we sum a constant at  $V$ . Integrated over the whole closed surface this constant is zero.

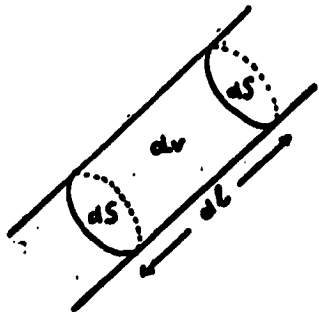


Figure 52. A flow tube with cross section  $dS$ .

For an arbitrary volume that can be thought of as built up from a collection of globules of various magnitudes, this expression goes over into an integral (see (28) page 29)

$$V = \frac{\int_{vol} E^* \cos \theta \, dv}{4 \pi r^2} \quad (77)$$

We can also think of this potential as originating from a current dipole placed in the origin of the system of coordinates. Here also we deduce the expression (see page 26)

$$V_i = \frac{D_i \cos \theta}{4 \pi \sigma r^2} \quad (21)$$

We can identify  $D_i$  here, obviously, with the total dipole. By making (77) and (21) equal we obtain

$$\vec{D}_i = \sigma \int \vec{E}^* \, dv \quad (78)$$

If we substitute (15), we obtain

$$\vec{D}_i = \sigma \int \nabla \vec{V} \, dv + \int \vec{j} \, dv \quad (79)$$

The second term of the right-hand member will be scrutinized more closely.

In agreement with Gabor and Nelson, we assume that in the volume in question there is neither positive nor negative source, and hence that  $\text{div } \vec{j} = 0$ . This is valid because the flow lines along which the electricity moves must be closed. If we remove one of the flow tubes and perform the integration  $\int \vec{j} dv$  we see (fig. 52) that the volume  $dv$  of a length of the tube along  $d\vec{l}$  can be written as  $dv = dS \cdot dl$ , because  $dS$  is the surface of the cross section. If we consider that vector  $\vec{j}$  is always directed along the line of flow, then

$$\int \vec{j} dv = \int j dS d\vec{l} \quad (80)$$

Here  $j dS$  is the amount of electricity that flows through the tube per 81 unit of time; hence, the current intensity. This magnitude, considering that  $\text{div } \vec{j} = 0$  is constant so that

$$\int j dS d\vec{l} = j dS \int d\vec{l} \quad (81)$$

The last integral is zero, because the lines of current flow are closed, as noted above. Thus

$$\int \vec{j} dv = 0 \quad (82)$$

We now write the first term, in analogy to what we determined in the first calculation

$$\oint_{\text{vol}} \vec{\nabla} V dv = \oint_{\text{opp}} V d\vec{S} \quad (83)$$

(Gauss) (equations (73) and (75a) so that finally we obtain

$$\vec{D}_1 = \sigma_{opp} \int \vec{v} d\vec{s} \quad (76)$$

the same result as that of Gabor and Nelson (see page 79).

What are the consequences of the above calculations for the practice of vectorcardiography? It is true that the result seems very attractive, but strictly speaking we have hardly made any progress because with these equations our point of departure has always been a homogeneous body, while a man is much less than homogeneous. It is possible to think up various arguments for leaving out of consideration this human lack of homogeneity. This implies however that only an "effective dipole" can be sought, which we must assume to be present somewhere in the body, and it also implies that the phenomena on the surface of the body must correspond to it as well as possible (see page 43). If we do this, we must apply the formulas above. It is obvious that in practice we are not able actually to determine an integral. We must therefore be content with the construction of a sum that more closely approximates actuality as the number of terms increases.

We shall now review some systems of vectorcardiography which attempt 82 to realize the goal in various ways, and which all have the common characteristic that they use more electrodes than the minimum of four<sup>1</sup> which was necessary according to the previously discussed dipole approximation, for determination of the three components of the heart vector.

#### 1. The BW system (Burger)

First we mention an attempt to combine two earlier systems. One system consists of three limb electrodes: right arm (R), left arm (L) and left leg (F) and a fourth electrode on the back (W) according to Wilson's system

<sup>1</sup>I.e., three independent leads.

(see page 33). The second system also makes use of the extremities, but the fourth electrode is on the chest (B). For the sake of convenience we designate the respective systems RLFW and RLFB.

If we consider these possibilities separately, we can anticipate flaws in each. To obtain an overall qualitative impression about this we recall that the heart is rather at the front of the thorax and this will certainly not lead to accurate measurements taken from the thorax. This means that part of the electric activity that is at the front of the heart influences electrode B more, i.e., it will impart to B a greater difference in potential with respect to the other electrodes. The reverse will naturally be the case with the electrode on the back, though to a lesser degree because the relative difference of distance between the front and back of the heart and W is less.

We can attempt to avoid this shortcoming, which is a result of the finite extensibility of the heart (better, the great spread of the dipole surface, see Chapters 15 and 16) by combining both systems and thereby providing each with one or another weight factor as expressed below

$$\alpha ( RLFW ) + \beta ( RLFB ) \quad (84)$$

whereby

$$\alpha + \beta = 1 \quad (85)$$

A correct choice is  $\alpha = 1/3$ ,  $\beta = 2/3$ . In this connection it must 83 be noted that the relationship between  $\alpha$  and  $\beta$  is not especially critical if RFLW and RLFB are well founded systems, and even that they might be selected with complete arbitrariness since both systems independently would yield

identical results. (In that case the dipole approximation must be assumed to be generally valid and thus combinations in general would be without meaning).

In practice the procedure has been as follows. In the first place the coefficients of the RLFW system are adapted as closely as possible to those of RLFB. This is done on the basis of the not exactly valid dipole approximation, and by means, of the method described in Chapter 10, 2. Briefly reviewed: when the locations of four electrodes (here R, L, F and B) are determined in the image space by model measurements, it is possible, applying a cancellation experiment, to determine which linear combination (equations (56) and (57)) of the three lead vectors yields the lead vector of the fifth

electrode (W). Different results (fig. 53) are obtained for different test individuals, and these results are averaged. The resulting electrode positions are shown in figure 54. Next we determine to what extent these systems overlap or, better, what differences they exhibit. It appears that the differences in the vectorcardiograms thus produced are not particularly large, but insofar as they are present they afford occasion for a direct but not especially critical choice of  $\alpha$  and  $\beta$ . The two systems (84) must now be combined according to the still rather vague formula (84). We shall explain it in more detail. In accordance with the dipole approximation we can write a set of three equations for each system, whereby the three components of the heart vector are expressed in the three independent potential differences, e.g. for RLFW

$$X_a = a_{11}V_{WR} + a_{12}V_{FR} + a_{13}V_{LR} \quad (36)$$

and similarly for  $Y_a$  and  $Z_a$

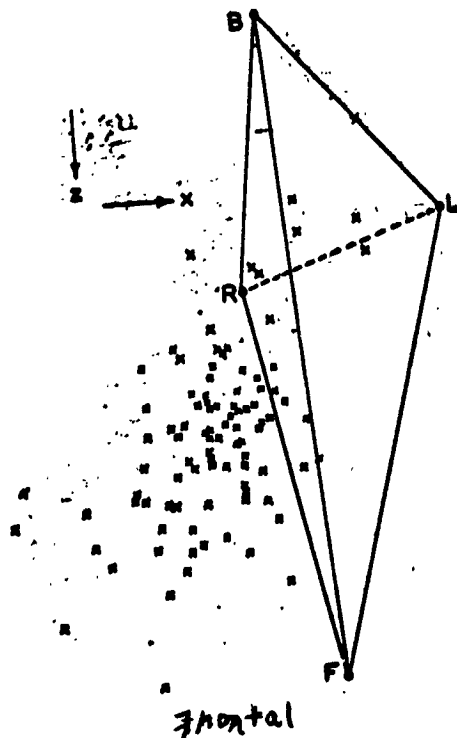


Figure 53. Measured results from determination of the location of W' in the image<sup>#</sup>space.  
^

Similarly for RLFB

$$X_b = b_{11}V_{BR} + b_{12}V_{FR} + b_{13}V_{LR} \quad (87)$$

and similarly for  $Y_b$  and  $Z_b$ .

The combination is then written as

$$\begin{aligned} X &= \alpha X_a + \beta X_b \\ Y &= \alpha Y_a + \beta Y_b \\ Z &= \alpha Z_a + \beta Z_b \end{aligned} \quad (88)$$

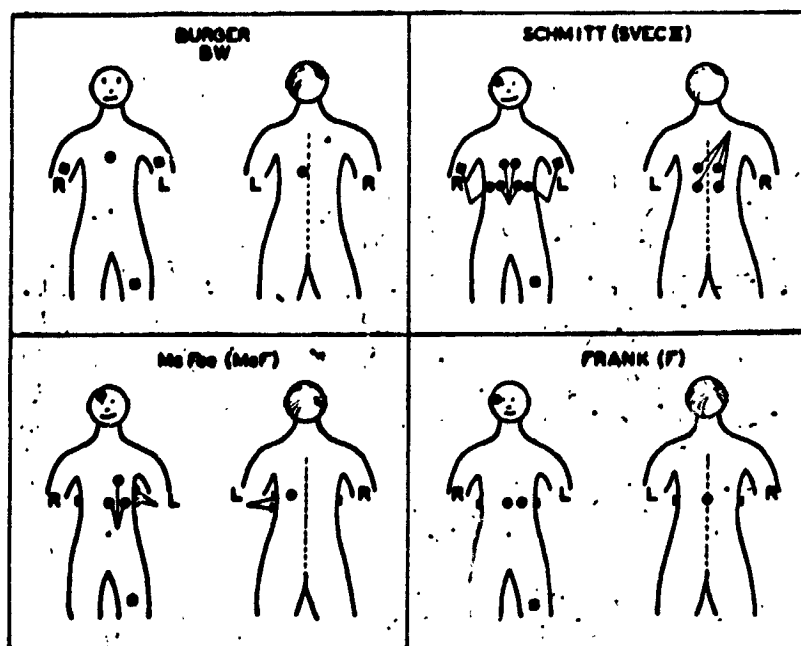


Figure 54. Locations of the electrodes in the four vectorcardiographic systems described.

We see from this that each component of the heart vector is no longer determined by three potential differences, but rather by four. While an initial attempt is reported, to become more independent of the position of the dipole effect by use of more electrodes than the minimal four, a similar effort with respect to the heart as generator of the electric phenomena can be imagined. We designate this a "coordinated system."

## 2. The F system (Frank)

This one which is also a "corrected system" uses seven electrodes (see figure 54). Whereas in the B system measurement on a model which is not homogeneous are used, in combination with results from experiments with test

individuals, Frank based his work entirely on measurements on a homogeneous model. In this model the location of the dipole, the artificial heart, varies while for each case extensive measurements of the dipole distribution on the surface are made. From them a system is developed whereby the electrodes are placed in such a way that the effect of a displacement of the dipole on the difference of potential between the electrodes is as slight as possible. Thus each component of the heart vector is formed by a linear combination of six differences of potential (indicated by Frank in the form of a system of resistors, as well as by coefficients).

### 3. The S system (Schmitt)

A method which is not substantially different from that of Frank is given by Schmitt, who also based his system on measurements on a homogeneous model. The number of electrodes is expanded here to fourteen (fig. 54).

Some are combined as a group by means of resistors in such a way that individual differences of position of the heart and the dipole distribution are averaged out. Moreover the way in which the electrodes are located recalls to a certain extent the old desire to obtain the three components of the heart vector directly "In the form of a voltage" (we bear in mind the fact that the unit of a dipole moment is  $V\text{cm}^2$ !) This now occurs, in 186 contrast to earlier systems (see page 134), in a way that is soundly based in physics.

### 4. The M system (McFee)

We shall here, as in the first case, treat the basis of the system rather extensively. McFee used the law of reciprocals, already established long ago by Helmholtz. We review the process of this calculation, whereafter the result will be adapted to the problems of vectorcardiography.

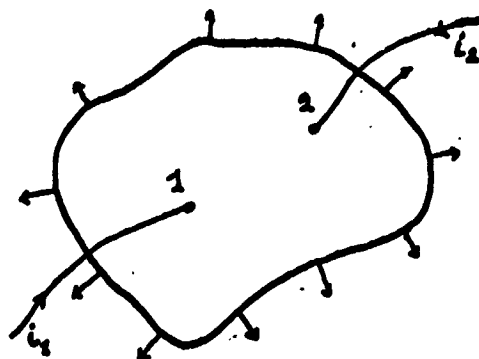


Figure 55. Two punctate positive current sources in a homogeneous medium.

We use Green's theorem which for any function  $\varphi$  and  $\Psi$  is

$$\int_{\text{vol}} [\varphi(\vec{r}) \nabla^2 \Psi(\vec{r}) - \Psi(\vec{r}) \nabla^2 \varphi(\vec{r})] dV = \int_{\text{surf}} [\varphi(\vec{r}) \vec{\nabla} \Psi(\vec{r}) - \Psi(\vec{r}) \vec{\nabla} \varphi(\vec{r})] \cdot d\vec{S} \quad (89)$$

or, in another notation

$$\int_{\text{vol}} [\varphi(x,y,z) \nabla^2 \Psi(x,y,z) - \Psi(x,y,z) \nabla^2 \varphi(x,y,z)] dV = \int_{\text{surf}} \left[ \varphi(x,y,z) \frac{d\Psi(x,y,z)}{dn} - \Psi(x,y,z) \frac{d\varphi(x,y,z)}{dn} \right] dS \quad (90)$$

in which  $\nabla^2$  is the Laplace operator and  $n$  is the normal of the closed surface over which integration is to be performed, positive at the outside.

In the following  $\varphi(x, y, z)$  and  $\Psi(x, y, z)$  are potentials.

We consider a closed surface (fig. 55) within which there is a homogeneous conducting medium. We imagine two point sources, 1 and 2. From these sources there is delivered current  $i_1$  and  $i_2$ , provisionally positive and of any magnitude, each leading to a potential field  $V_1$  and  $V_2$ . To prevent an accumulation of electric charge, the currents which are

introduced must be carried off. We assume that this discharge occurs /87  
uniformly over the entire surface. This implies that at the surface  
 $dV/dn \neq 0$ . Particularly we select  $dV/dn = \text{constant}$  for both  $V_1$  and  $V_2$ .  
(How this can be accomplished in actuality is not the question: we use this  
only as a mathematical artifice). On the superposed potential fields excited  
by  $i_1$  and  $i_2$  we adapt Green's theorem. According to (90)

$$\int_V (V_1 \nabla^2 V_2 - V_2 \nabla^2 V_1) dV = \int_S (V_1 \frac{dV_2}{dn} - V_2 \frac{dV_1}{dn}) dS \quad (91)$$

must then be valid.

In the lefthand member

$$\int_V V_1 \nabla^2 V_2 dV = 0 \quad (92)$$

will be true everywhere except for a volume element ( $\Delta V_2$ ) in the immediate  
vicinity of source 2. We now introduce the following notation:  $(V_1)_2 \equiv$  the  
potential at point 2, caused by the current introduced at 1, and similarly  
for other combinations. We recall that  $V_1$  is practically unchanged in  $\Delta V_2$ ,  
so that

$$\int_{\Delta V_2} V_1 \nabla^2 V_2 dV = (V_1)_2 \int_{\Delta V_2} \nabla^2 V_2 dV = - (V_1)_2 \frac{i_2}{\epsilon} \quad (93)$$

We obtain for the lefthand member

$$\int_V (V_1 \nabla^2 V_2 - V_2 \nabla^2 V_1) dV = - (V_1)_2 \frac{i_2}{\epsilon} + (V_2)_1 \frac{i_1}{\epsilon} \quad (94)$$

From the righthand member of (91) we have  $\int_S V_1 \frac{dV_2}{dn} dS$ . As previously set as a condition,  $dV_2/dn = \text{constant}$ . Thus this term may be used for the integral. Divide by--and multiply with--S then, as readily can be seen, yields

$$\int_S V_1 \frac{dV_2}{dn} dS = \frac{dV_2}{dn} S \frac{\int_S V_1 dS}{S} = -\frac{i_2}{\sigma} \bar{V}_1 \quad (95)$$

in which  $\bar{V}_1$  is the average of the surface potential caused by source 1. /88

We thus determine the whole righthand member

$$\int_S \left( V_1 \frac{dV_2}{dn} - V_2 \frac{dV_1}{dn} \right) dS = -\frac{i_2}{\sigma} \bar{V}_1 + \frac{i_1}{\sigma} \bar{V}_2 \quad (96)$$

(94) and (96) combined yield the equation

$$(V_1)_2 i_2 - (V_2)_1 i_1 = -i_1 \bar{V}_2 + i_2 \bar{V}_1 \quad (97)$$

This result, as the result of a general example, will be adapted to a more specific case, from which again we are able to adapt to vectorcardiography.

We again consider a homogeneous conducting body with four current sources, A, B, C, and D rather than two. If we introduce a current  $i_1$  at A and carry it off at B while we do the same with  $i_2$  at C and D, we must assume the body to be insulated and thus abandon the rather artificial method of conducting the current over the surface (fig. 56). If we now consider points A and C first alone (considering that at these points only current can be introduced into the body) we shall once more make use of the assumption  $dV/dn = \text{constant}$ . In accordance with (97) we can write

$$(V_C)_A \cdot i_1 - (V_A)_C i_2 = i_1 \bar{V}_C - i_2 \bar{V}_A \quad (98)$$

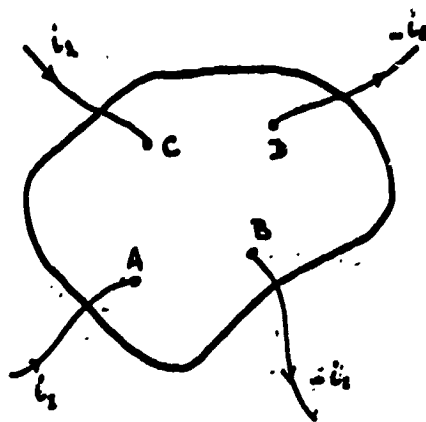


Figure 56. Two positive and two negative current sources in the homogeneous medium.

A similar expression is valid for points B and C with the understanding that for the current at the location of point B, the value  $-i_1$  must be assigned

$$(V_C)_B i_1 - (V_B)_C i_2 = -i_1 \bar{V}_C - i_2 \bar{V}_B \quad (99)$$

If we add (98) and (99), then

$$(V_C)_{AB} i_1 - (V_{AB})_C i_2 = -i_2 (\bar{V}_A + \bar{V}_B) \quad (100)$$

in which  $(V_C)_{AB}$  represents the difference of potential between points /89  
A and B, caused at C by  $i_2$ , while  $(V_{AB})_C$  is the potential at C, the result of  $i_1$  at A and  $-i_1$  at B. (Since the conducting wires can carry off the charge, we can replace the artificial  $dV/dn = \text{constant}$  with  $dV/dn = 0$ .)

It can readily be seen that a similar formulation appears when instead of C the point D is taken (whereby as previously we substitute  $i_2$  for  $-i_2$ ) thus

$$(V_D)_{AB} i_1 + (V_{AB})_D i_2 = i_2 (\bar{V}_A + \bar{V}_B) \quad (101)$$

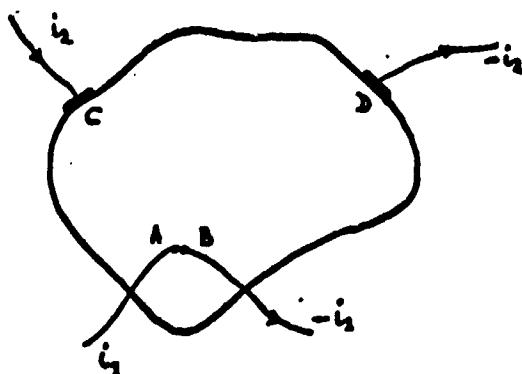


Figure 57. Points A and B are brought near each other to a dipole. C and D are placed on the surface.

We can now add (100) and (101) with the result

$$(V_{CD})_{AB} i_1 = (V_{AB})_{CD} i_2 \quad (102)$$

where of course  $dV/\bar{a}n = 0$ .

This is the principle of reciprocals of Helmholtz, symmetrical at AB and CD.

Following McFee we shall adapt this result to vectorcardiography. We assume points A and B to be so close together that we can speak of a current dipole. At the same time we permit C and D to separate, and especially we select locations for them on the body surface (fig. 57). We now allow a current to pass from A to B and from C to D and apply the theorem of reciprocals to our case, adapted to the indicated situation. The difference of potential between A and B,  $(V_{CD})_{AB}$  we now express as the field density  $(\vec{E}_{CD})_{AB}$  between A and B, scalar product of vector  $\vec{AB}$ .

For the voltage between C and D we use the same expression that we had before for the difference of potential between two electrodes, caused by a dipole somewhere in the body, namely the scalar product of lead vector  $\vec{abc}$  with dipole moment  $\vec{XYZ}$  (equation (40)): thus /90

$$(\vec{E}_{CD})_{AB} \cdot \vec{AB} \cdot i_1 = \vec{abc} \cdot \vec{XYZ} \cdot i_2 \quad (103)$$

Since AB is a current dipole and  $\vec{XYZ}$  is a voltage dipole, the second member becomes

$$\vec{abc} \cdot \vec{XYZ} \cdot i_2 = \vec{abc} \cdot \frac{\vec{D}_1}{\epsilon} \cdot i_2 = \vec{abc} \cdot \frac{\vec{AB} \cdot i_1}{\epsilon} \cdot i_2 \quad (104)$$

(see page 26). Since (104) is valid for any  $\vec{AB}$ , we can write

$$\epsilon (\vec{E}_{CD})_{AB} = i_2 \cdot \vec{abc} \quad (105)$$

for (103), or

$$(\vec{J}_{CD})_{AB} = i_2 \cdot \vec{abc} \quad (106)$$

This is an important result, for we now see that, aside from one constant factor, the current intensity and the lead vector are the same. We adapt this to the following example. If we assume that a current  $i$  is carried by the human body from the left arm to the right, then the current field produces at an arbitrary place in the body the lead vector for electrodes R and L when a dipole is so positioned in the same place that it gives rise to a

difference of potential between R and L. The current field in this association is designated lead field.

We may now observe the following. Given two electrodes inside the body wherein a dipole is placed. If we assume that the voltage between the electrodes need not be dependent upon the location of the pole, this statement is equivalent to: the current field ("lead field") in the region, as a result of a current carried from one electrode to the other must be homogeneous.

Generalizing for more electrodes: for an ideal lead system there must be such a combination of differences of potential between various electrodes that at the location of the heart the place of the dipole has no role. Using the reciprocals formula we can also state the case thus: a combination of currents controlled by the various electrodes must deliver a homogeneous lead field at the location of the heart.

With this remark we return in fact to the Gabor and Nelson formula /91 (76) which in principle presents a method for determining the total dipole moment independent of the location of the dipole.

Experimentally the problem can be attacked this way.--Control an electric current by a plurality of electrodes in a model and vary the positions of the electrode in such a way that at the location of the heart a homogeneous lead field is developed. The electrodes which rationally meet this requirement, with the correct coefficients will--conversely--be usable for determination of the total dipole moment.

McFee did not work with an electric current, but with a fluid flow which he allowed to pass over granules of  $\text{KMnO}_4$  in a glass model (originally two-dimensional and heterogeneous, later three-dimensional and homogeneous). The lead field here was represented by paired lines.

The result of McFee's experiments was a system of nine electrodes.  
Figure 54 page 84 indicates how they are distributed over the body.

In all this we must remind ourselves that in principle it is impossible to meet the requirement of a homogeneous lead field at the location of the heart, caused by a current controlled by a plurality of electrodes. Blood, myocardium and other kinds of tissue do not all exhibit the same conductivity. This implies a "refraction" of the current line and thus in transition from one medium to another there is a bend, therefore homogeneity exclusively [is required].

This means that we must assume the effective dipole (i.e., the dipole that produces the field as satisfactorily as possible, measured at the body surface) to be located in a "heart cavity" filled with a homogeneous conducting medium. What conductivity must we select? A firm choice is not to be made, and therefore with there remains an essential imponderable in the problem.

This does not mean that it is not desirable to find a medically usable system that is grounded in physics as solidly as possible, but it is well to bear this principle<sup>10</sup> limitation in mind.

In this connection we emphatically warn against the numerous, but inaccurate that are advanced, as though all difficulties and uncertainties concerning the electric activity of the heart and the leads were specific to vectorcardiography. Of course these problems are basic to electrocardio<sup>9</sup>graphy itself, though less conspicuously so in that field.

Still one more system must be mentioned, namely that of Rylant. /92  
This system uses 72 electrodes which, to meet the requirement of practical usability, are held in a flexible rubber vest so that patients with different

body measurements can easily be tested. The electrodes are interconnected by a passive network of resistors. Although the author states the grounds

inadequately in his publications, he suggests that in the theoretical background use can be made of the properties of nonEuclidian space. As a consequence of the complexity of the problem, it is not yet possible to evaluate this system.

## CHAPTER 12.

### Subjective and Objective Comparison of Vectorcardiography Systems. Compromise in Vectorcardiography

In all possibilities for the design of vectorcardiography systems it 193 is of course of very great importance to establish a criterion whereby it is possible to estimate the extent to which the image, the vectorcardiogram obtained by a given system, actually is representative of electric events in the heart, and also to estimate its practical value. In the preceding chapters it has become evident that the ideal system does not exist. It is not possible to judge a system alone and we must therefore proceed by no other means than comparison, that is to say, find out to what degree the results of two systems are in agreement.

We shall apply two working methods, subjective and objective.

#### 1. Subjective comparison

This method is very simple in nature and is based on nothing other than the assigning of an evaluation in figures from 0 to 10. The procedure is as follows.

Two vectorcardiograms for one patient, obtained with the two systems that are to be compared, are judged by three individuals who express their evaluation concerning correlation by means of a figure. If the evaluations differ only slightly, they are averaged and then rounded off to a whole number. If there is a greater deviation between evaluations than the doubt which the compilers felt in making their judgments, then a consultation is held, to reach a consensus. If this is done for a great number of patients, a reasonable impression is finally obtained on the measure in which two systems correspond.

The subjective element in this method of comparison lies primarily in the fact that special emphasis will be laid on characteristics that are of diagnostic importance, rather than on purely geometric data. Thus for example in frontal and horizontal projection the proportion of maximum deflection /94 in the x direction on either side of the center of the loop is important (so-called right-left ratio). Furthermore, for example, the direction of the circulation is of essential diagnostic significance, whereas on the contrary the size of the loop is of less interest.

If in the comparison of a new series of vectorcardiograms the recollection of a preceding one is kept in mind, the evaluator attains a true continuity in the meaning of the figures.

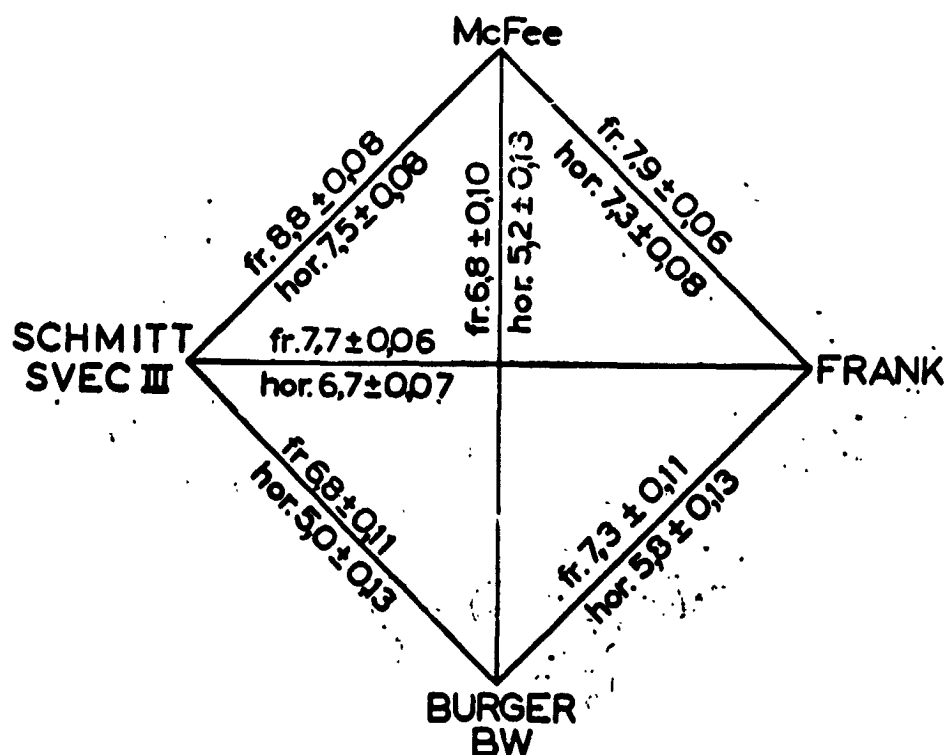


Figure 58. Diagram showing the subjective comparison of four vectorcardiography systems.

The result of a comparison (in approximately 175 cases) of the four corrected systems described in the preceding chapter is shown in figure 58. Only the frontal and horizontal projections are presented (fig. 25, page 34 and fig. 26, page 35). This has its basis in the nature of the instrumentation: the vectorcardiograph was not yet developed for recording of the sagittal projection. (The compilers of the present schema are aware of course that the three components were not evenly evaluated.)

In each evaluation the average standard error is shown, determined in the customary way by application of the formula 95

$$\text{standard error} = \sqrt{\frac{\sum_1 (x_1 - \bar{x})^2}{n(n-1)}} \quad (107)$$

We observe the following in figure 58.

a. In the comparison of a pair of systems it appears that the frontal projection significantly is always judged more favorably than the horizontal. Decidedly this is due to the fact that the sagittal component was so strongly affected by the circumstance that in the Z direction ( fig. 25, page 34) the heart has relatively such large dimensions, as a consequence of which there is a great difference between an excitation at the front and one at the back of the heart. These geometric differences have less significance for the other two components.

b. The mutual correlation between the Schmitt, McFee and Frank systems is clearly better than between Burger's and the other three. The cause of this may be sought inter alia in the corresponding and/or differentiating starting points from which the respective systems developed. In Utrecht the

starting point was distinctly heterogeneous models and test individuals, whereas the other three systems are based on measurements on homogeneous models (however, see page 91).

c. All four systems ignore anisotropy in their calculations. Yet this phenomenon must play a not insignificant role, as measurements made in the course of years have indicated that the conductivity in the lengthwise direction of muscle is approximately 15 times that which obtains in the transverse direction. The way in which the anisotropy might be brought into the calculation is still so obscure that for the present it is disregarded.

## 2. Objective comparison

We indicated earlier (page 72) that if the dipole approximation is accurate, it must always be possible to establish a linear transformation which converts the two systems, each with four or more electrodes, into each other. If we drop the dipole approximation and proceed on the basis of the Gabor-Nelson theory, in systems with four or more independent leads (thus systems "corrected" for the extent and displacement of the dipole layer, see Chapter 15 and 96 16), then in principle we will no longer anticipate such a linear relationship.

We run into trouble especially when we calculate a transform matrix for one patient and attempt to adapt it to another. Results are better if we try to develop an average transformation from a broader material. A similar linear relationship then seems to have a better right to existence, as it concerns the comparison of systems averaged over many individuals. It is for this reason that we shall consider somewhat more in detail the general method than can be used in seeking this average transformation.

We consider, as in Chapter 10, 3 page 73, two vectorcardiograms (fig. 59) representing two systems L and K. On loop L we select one or another point and

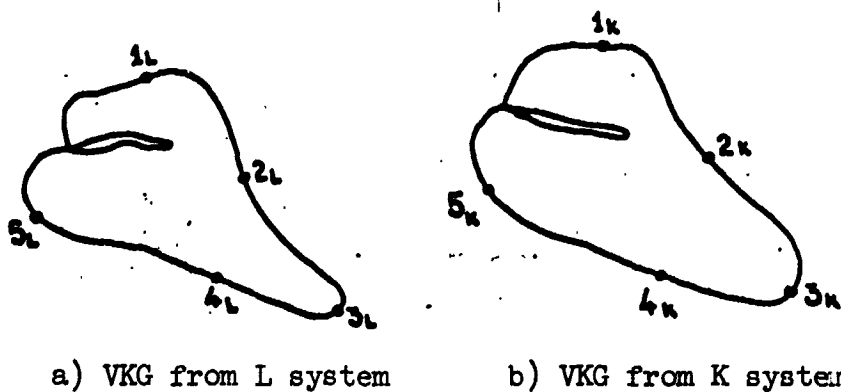


Figure 59. One projection is representative of the entire loop.

then the isophasal point on loop K, and we designate the coordinates  $X_L, Y_L, Z_L$  and  $X_K, Y_K, Z_K$ , then with assumed accuracy of the dipole approximation:

$$\begin{aligned} X_K &= p_x X_L + q_x Y_L + r_x Z_L \\ Y_K &= p_y X_L + q_y Y_L + r_y Z_L \\ Z_K &= p_z X_L + q_z Y_L + r_z Z_L \end{aligned} \quad (108)$$

wherein the 9 coefficients  $p_x, \dots, r_z$  are dimensionless magnitudes, /97  
dependent only upon the choice of the coordinate system and systems K and L. In the set of three equations  $X_K \dots Z_L$  are known while the coefficients are unknown. If we wish to calculate the latter we must expand the number of equations to at least nine, i.e., we must establish equation (108) for at least three isophasal pairs of points.

Instead of three, we can take five points on the loops and establish equations for each pair. We then have 15 equations for 9 unknowns. The problem is now overdetermined. In order in such a case to solve the system of equations we can use the method of least squares. But we go further and solve not

for one loop but collect a series of vectorcardiograms for a great number, e.g.,  $N^1$  patients so that we have  $3 \cdot 5 \cdot N$  equations for nine unknowns. The method of least squares gives us "average" values for  $p_x \dots r_z$ . (The calculations here required can only be done by a "highspeed" electronic computer.)

The result, written as:

$$\begin{aligned} X_{L \rightarrow K} &= p_{x(L \rightarrow K)} X_L + q_{x(L \rightarrow K)} Y_L + r_{x(L \rightarrow K)} Z_L \\ Y_{L \rightarrow K} &= p_{y(L \rightarrow K)} X_L + q_{y(L \rightarrow K)} Y_L + r_{y(L \rightarrow K)} Z_L \\ Z_{L \rightarrow K} &= p_{z(L \rightarrow K)} X_L + q_{z(L \rightarrow K)} Y_L + r_{z(L \rightarrow K)} Z_L \end{aligned} \quad (109)$$

shows with which coefficients we multiply the three coefficients of the L system in order, after summing, to obtain average components which we would otherwise be able to obtain "directly" from the K system. In other words, the transformation affords the possibility of simulating the vectorcardiograms of the K system by means of the averaged L system.

Without going into the details of the method, we note here certain results which can be taken as examples: the transformations from the M to the S system and from the B to the M system.

---

<sup>1</sup>Calculations in Utrecht were based on 169 cases, 41 normal individuals and 128 patients with heart defects.

$$X_{M-S} = +(0,86 \pm 0,010)X_M + (0,01 \pm 0,013)Y_M - (0,09 \pm 0,010)Z_M$$

$$Y_{M-S} = +(0,01 \pm 0,005)X_M + (0,88 \pm 0,006)Y_M + (0,00 \pm 0,005)Z_M$$

$$Z_{M-S} = +(0,21 \pm 0,016)X_M + (0,13 \pm 0,021)Y_M + (1,05 \pm 0,016)Z_M$$

(110)

$$X_{B-M} = +(0,71 \pm 0,013)X_B + (0,22 \pm 0,028)Y_B + (0,24 \pm 0,019)Z_B$$

$$Y_{B-M} = +(0,05 \pm 0,007)X_B + (0,97 \pm 0,011)Y_B - (0,17 \pm 0,010)Z_B$$

$$Z_{B-M} = -(0,39 \pm 0,021)X_B + (0,60 \pm 0,033)Y_B + (0,92 \pm 0,030)Z_B \quad (111)$$

How do we judge such a transformation? Naturally we must first ascertain what we expect, or rather what we want to have appear. That is, as remarked previously, the identical transformation given by the matrix coefficients (page 72).

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (112)$$

In general the coefficients  $p_x \dots r_z$  do not vary too much from those of the identical transformation (see (110) and (111)), but for certain individuals the correspondence of (110) and (111) is poor. Not only can we draw the conclusion from this that the systems are clearly unlike, but it is even possible

to obtain an indication wherein the systems differ. Thus the large coefficient  $q_{z(B \rightarrow M)}$  in (111) indicates that a larger Y component in the B system will yield a larger Z component in the M system, or, speaking in geometrical terms, a downward-directed loop in the B system corresponds to a somewhat larger backward (or smaller forward)-directed loop in the M system.

In the calculations that lead to a transformation we recall that the "absolute size" of the loop plays a role which becomes evident in the measurement of the coordinates of the points. Just as in electrocardiography, <sup>/99</sup> this absolute magnitude is generally unimportant in vectorcardiography. Nevertheless a source of error is concealed here, for the ignoring of differences in magnitude as is the practice in the clinic has no objective analogs. Therefore it is necessary to adapt the scales of the different systems to each other as closely as possible. In addition an average measure of the magnitude of the loop is introduced, characterized by:

$$\bar{H} = \sqrt{\frac{\sum x^2}{N} + \frac{\sum y^2}{N} + \frac{\sum z^2}{N}} \quad (113)$$

whereby all five points of all N systems are summed. For each system we make  $\bar{H}$  equal to one of the above selected standards (for which the B system is used). Thus we have a possibility of adapting the systems to each other in magnitude by means of a correction factor.

With knowledge of the transformation we have remarkably little quantitative insight, however, into the measure in which the two systems agree or differ (because a datum of this kind is of course implicit in location). For the difference between system L and the system L-K obtained from L by

transformation we therefore define the "distance" which is developed by the transformation:

$$(L - L \rightarrow K) = \sqrt{(x_{L \rightarrow K} - x_L)^2 + (y_{L \rightarrow K} - y_L)^2 + (z_{L \rightarrow K} - z_L)^2} \quad (114)$$

wherein

$$\begin{aligned} (x_{L \rightarrow K} - x_L) &= (p_{x(L \rightarrow K)} - 1)x_L + q_{x(L \rightarrow K)}y_L + r_{x(L \rightarrow K)}z_L \\ (y_{L \rightarrow K} - y_L) &= p_{y(L \rightarrow K)}x_L + (q_{y(L \rightarrow K)} - 1)y_L + r_{y(L \rightarrow K)}z_L \\ (z_{L \rightarrow K} - z_L) &= p_{z(L \rightarrow K)}x_L + q_{z(L \rightarrow K)}y_L + (r_{z(L \rightarrow K)} - 1)z_L \end{aligned} \quad (115)$$

For this distance we obtain an average value by which  $x_L^2$ ,  $y_L^2$ , ... etc. in

(114), average  $\frac{\sum x_L^2}{N}$ ,  $\frac{\sum y_L^2}{N}$ , ... etc. are summed over all five N points for

substitution. If we then divide by  $\bar{H}$ , we have the average relative distance  $\delta$  which makes possible the numerical expression of the difference between 100 two systems

$$\delta = \frac{(L - L \rightarrow K)}{\sqrt{\frac{\sum x^2}{N} + \frac{\sum y^2}{N} + \frac{\sum z^2}{N}}} \quad (116)$$

(no confusion can arise over the denominator which is the same for the two systems).

We can now obtain still another expression which states how far the L vectorcardiogram transformed to K agrees with the true K system. In other words, how successful we have been in imitating the K system with the L system transformed to K. For this we compare  $L \rightarrow K$  with K itself. In a way similar to

that above, we determine a relative magnitude for this deviation of L-K:

$$\Delta = \frac{\sqrt{(x_K - x_{L \rightarrow K})^2 + (y_K - y_{L \rightarrow K})^2 + (z_K - z_{L \rightarrow K})^2}}{\sqrt{\frac{\sum x^2}{N} + \frac{\sum y^2}{N} + \frac{\sum z^2}{N}}} \quad (117)$$

The denominator is equal to  $\bar{H}$  as defined in (113). In this expression we again substitute for  $x_K^2 \dots \frac{\sum x^2}{N}$  etc.

We present some numerical examples for values  $\delta$  and  $\Delta$  to indicate the order of magnitudes.

| Transformation | $\delta$ | $\Delta$ |
|----------------|----------|----------|
| B-M            | 0.35     | 0.44     |
| M-S            | 0.22     | 0.31     |

(118)

What conclusion must we draw from this concerning the practical value of the transformation? Let us consider what can be learned from  $\delta$  and  $\Delta$  in extreme cases.

Ideally  $\delta = 0$  and  $\Delta = 0$ . That is to say, there is no difference between systems L and K, for example. The transformation is identical and thus /101 need not be worked out. Furthermore, deviation is zero--an inconceivable situation in actuality--which is to say that the results of the L and K system are exact at all times<sup>1</sup>.

<sup>1</sup>The latter is practically not controllable, since no vectorcardiograph is equipped to record two or more systems simultaneously. Since the electric activity of the heart in two successive beats is never entirely the same the deviation introduced by this phenomenon cannot be eliminated.

A small  $\delta$  with large  $\Delta$  must mean that the distance covered by the transformation is small with respect to deviation. It is obvious that then the transformation has no single meaning. This contrasts with the situation in which  $\delta$  has a specific value and the corresponding  $\Delta$  is small. The transformation can then be fruitfully applied, to obtain with the electrode placement of one system the results of another system of vectorcardiography.

The table presents values for  $\delta$  and  $\Delta$  which do not differ greatly. In view of the subjective element concealed in a statement in a similar case with little evidence, we assume here that the transformation is meaningful enough (the reader is well aware that the "origin" is not exact, and that physics however exact it also may seem as a science cannot exist without human interpretation). Experimental physics is often squarely between the subjective choice of method and the evaluation of the final result.

This conclusion is supported by what a "subjective comparison" yields. If specifically we compare the transformed system with the system into which it is transformed, the evaluation of the one generally determined turns out to be more favorable than when we consider the two original vectorcardiograms, as demonstrated in (119) in which the results of the B and M system are reported.

| transformation | $\delta$ | $\Delta$ | comparison | subjective evaluation |               |
|----------------|----------|----------|------------|-----------------------|---------------|
|                |          |          |            | frontal               | horizontal    |
| B-M            | 0.35     | 0.44     | B with M   | $6.9 \pm 0.1$         | $5.8 \pm 0.1$ |
|                |          |          | B-M with M | $7.9 \pm 0.1$         | $6.7 \pm 0.1$ |

(119)

Figure 60 presents several examples taken from actual practice. They are intended to illustrate the above discussion, for which reason

/102

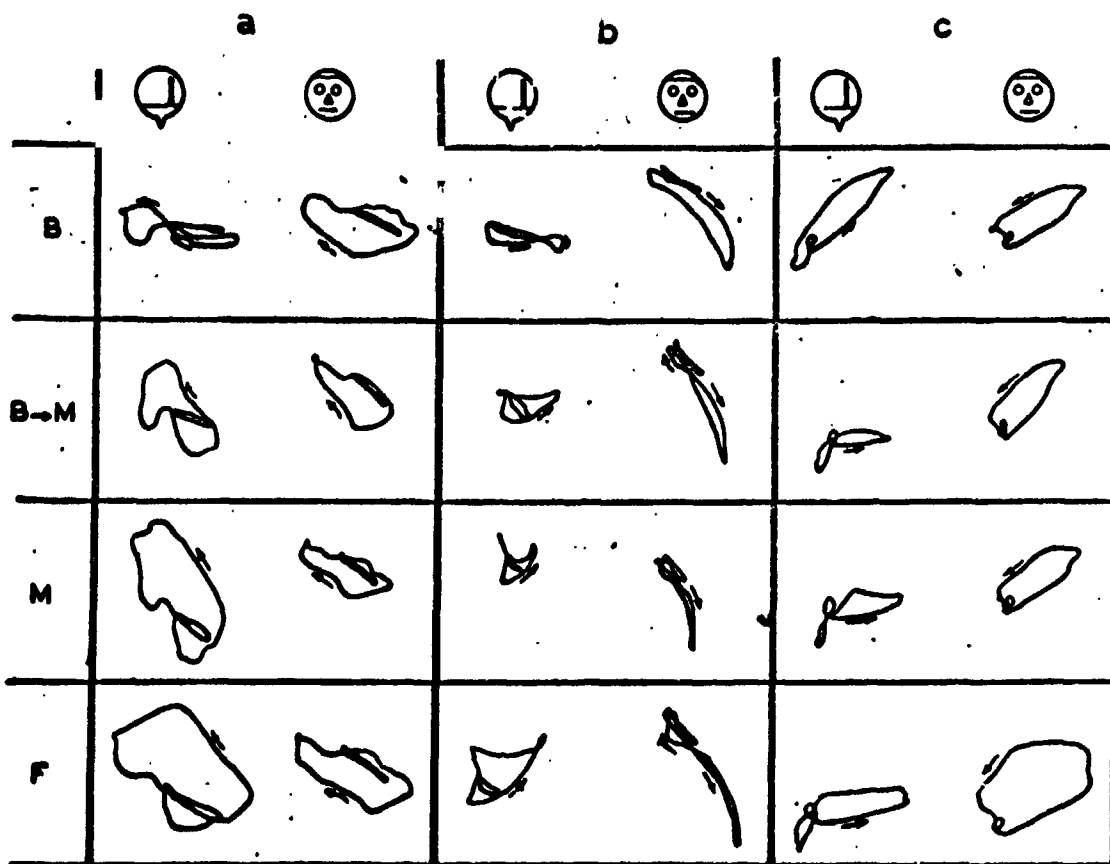


Figure 60. Three examples of successful transformations from B to M system.

satisfactorily tallying cases have been selected.

On page 73 we were concerned with the individual transformation. We calculated coefficients  $p_x \dots r_z$  from five pairs of isophasal points. The deviation in the result (hence the divergence of the transformed system from the system to which it was transformed) appears generally small with respect to distance  $\delta$  over which the transformation was made, as a result of the fact that the transform matrix was "fitted" to the individual. If we compare transformations of  $\delta$  for a greater number of individuals we find--unfortunately--great differences, even to a factor of five between maximum and minimum values. The large  $\Delta$  in the averaged transformation is thus not the immediate result of inaccuracy of the dipole approximation (for it is successful for one individual) but must stem primarily from the great variations between individuals.

What can we learn now from all these considerations concerning the different systems?

/103

First it has become clear, what physical basis has been selected, and what hypotheses were developed, though there remains some uncertainty in the final result, because our object is not purely physical in nature but also biological. The one who conducts electrocardiography by "cookbook recipe" is blind to what the problem involves. Vectorcardiography forces the one who applies it to be aware of the physical difficulties that are hidden in the method and which only come clearly to light when several systems are used.

Secondly, each of the four physically based systems which we have considered, with few exceptions, leads to the same clinical diagnosis. In this way they can be regarded as clinical equivalents.

The more immediate question is obvious. Should one system be selected and applied as the universally accepted one? All the confusion in the field of vectorcardiography would thus be relegated to the past. Comparison of vectorcardiograms made anywhere in the world would be possible.

Unfortunately psychology comes into the discussion with a sharp word when the choice of a system is involved, to such an extent that what seems to be simple is never brought to reality.

Is physics perhaps in a position to bridge the gap of opinions? When differing standpoints must be coordinated, it is usual to hunt for a mutually acceptable compromise. In terms of physics this means that system L ought to be transformed halfway to K, and system K ought to be half way transformed to L. In practice this can be effected simply by averaging the coefficients. We can average the coefficients of L into those of the transformation of L-K, in brief notation written as

$$L_{LK} = \frac{L + L \rightarrow K}{2} \quad (120)$$

and conversely

$$L_{KL} = \frac{K + K \rightarrow L}{2} \quad (121)$$

If we compare the two vectorcardiograms which have each undergone a half transformation step according to (120) and (121), correspondence is /104 certainly better than when a complete transformation is performed. Since the users of the two systems L and K each must adjust the respective system in a similar way, the results of both operations should be readable without much difficulty although each user still is able to retain his "own" electrode system. To generalize this thought by constructing an "average" system by a transformation of each vectorcardiography system is obvious. If the decision can be made to accept such a proposal, an important step will be taken in the direction of unified vectorcardiography.

Finally we must address a few comments to the matter of the linear transformation. If the average transformation has been calculated to convert system L to system K, the reverse operation from K to L is not yet known.

This can be appreciated in a one-dimensional case. When a great number of points  $X_K$  and  $X_L$  is given and an arbitrary point  $X_L$  must be transformed to K using  $X_{L \rightarrow K} = p_{x(K \rightarrow L)} X_L$ , the method of least squares yields  $p_{x(L \rightarrow K)}$  through

$$\sum x_K x_L = p_{x(L \rightarrow K)} \sum x_L^2 \quad (122)$$

For the reverse transformation  $X_{K \rightarrow L} = p_{x(K \rightarrow L)} X_K$  we find  $p_{x(K \rightarrow L)}$  from

$$\sum x_L x_K = p_{x(K \rightarrow L)} \sum x_K^2 \quad (123)$$

We see immediately that

$$P_x(L \rightarrow K) \neq \frac{1}{P_x(K \rightarrow L)} \quad (124)$$

Unfortunately it is not true that we can deduce the transformation  $L_1 \rightarrow L_2$  when we know transformations  $K \rightarrow L_1$  and  $K \rightarrow L_2$  of three systems, e.g.,  $K$ ,  $L_1$  and  $L_2$ .

This means that we must perform the complete calculation to find all /105 transformations between each pair of systems, and in each direction also.

## CHAPTER 13.

### The Pole Vector ( $\vec{P}$ )

Electrocardiography lacks methods for attempting to collect data, /106 as the case is in many other physical sciences. We obtain so much of the desired material that quantity is high over our heads and we cannot recover quality. Mathematics long ago devised ways of bringing some comprehensible order into the multiplicity of data by introducing averages, standard deviations, significance tests, etc.

A vectorcardiogram also contains in fact a very great amount of information--no matter how simple the appearance of a loop may be--and we should therefore readily find methods to characterize this information, particularly specific aspects of it, in a simple and visibly striking manner. This wish has led to the introduction of simple magnitudes which are specific for certain properties of the vectorcardiogram. Two of them, the pole vector ( $\vec{P}$ ) and the ventricular gradient ( $\vec{G}$ ) will be discussed in the present chapter.

As we noted previously (page 37) the direction of movement of the dot on the oscilloscope screen at the time of "writing" the vectorcardiogram is an important datum in making a diagnosis. This is especially true of the frontal projection. By using periodic beam interruptions it is possible to determine this direction on the record (see fig. 61): by depression of the electron beam the intensity is diminished little by little so that the "point" of the dot indicates the direction of travel (page 37). Difficulties arise, however, because in the frontal projection the loop is seen "on edge." A very flat loop

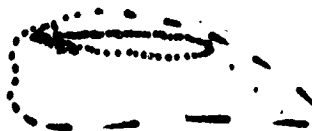


Figure 61. A vectorcardiogram with time marking. The direction of travel follows from the form of the dots.

is developed, rather than an open one, or a figure eight in which one part moves clockwise and the other counterclockwise. The desire to have a clear /107 criterion in such cases also, wherein direction of circulation of the vector or, more generally, the spatial orientation of the plane of the loop is discounted, has been the occasion for the introduction of the "pole vector" concept.

Let us start with the assumption that the loop (we always mean that of the QRS complex) lies entirely in a flat plane. We then indicate the position of this plane by the normal thereof. The direction in which this normal points follows from the direction of circulation of the vectorcardiogram with application of the well known law of the screw. The particular feature that we want to characterize can thus easily be expressed by the position of this "pole vector."

It seems less simple to find a similar criterion for the case in which the loop is not in a flat plane but twists arbitrarily through space. As we shall prove, it is still possible in that case to define a pole vector.

Through the closed curve--as the QRS loop of a vectorcardiogram usually is--we provide an arbitrarily curved surface ( $S$ ). By means of it we are able to determine a definition of the normal of the "surface of the loop." We divide  $S$  into surface elements  $dS$  which are so small that we consider each

individually as a plane. Each of these elements is to be represented by a vector, perpendicularly, with a length parallel to the surface (sic) and pointing in the direction which as we indicated above follows the law of the right-hand screw applied to the vectorcardiogram. All vectors that we can construct in this way are (vectorally) added. The result is then  $\vec{P}$ . Within the limit this becomes an integral:

$$\int d\vec{S} = \vec{P} \quad (125)$$

We note that this expression is not dependent upon the selection of the supplied surface, but is exclusively determined by the contour thereof, hence by the vectorcardiogram. We supply a second surface  $S'$  through the curve, so that a sort of lenticular enclosure is developed. We also divide  $S'$  into elements to develop  $\int d\vec{S}'$  in analogy to (125) with the understanding that we oppose the direction of  $d\vec{S}'$  on  $S'$  to the direction of  $d\vec{S}$ .

If we inquire about the sum of the two integrals, we encounter a known fact: the vectoral integral of the superficial elements of a closed /108 surface is zero (see note on page 79). Thus for unidirectional  $d\vec{S}$  and  $d\vec{S}'$ :

$$\int d\vec{S} = \int d\vec{S}' = \vec{P} \quad (126)$$

$\vec{P}$  is thus exclusively dependent upon the contour of the surface, hence upon the vectorcardiogram.

How do we determine  $\vec{P}$  in practice? It can be done very simply, because the three components of  $\vec{P}$  are exactly equal to the surfaces of the corresponding projections of the vectorcardiogram. It can be readily seen that this is so for a flat surface. The surface of the projection of the zy surface and  $P_x$  are both determined by the surface with respect to  $P$ , multiplied by  $\cos \alpha$

whereby  $\alpha$  is the same in both cases. That the statement is also valid for a curved surface is a consequence of the fact that we can think of it as composed of small flat surfaces.

The surfaces of the three projections are determined by means of a planimeter. With this in view, it is important that the vectorcardiograph have three oscilloscope screens.

Experience has shown that diagnostically the magnitude of the loop has less significance than the position of the loop in space and the direction in which it moves. This has led to the following geometric representation and clinical method: We cause the pole vector to originate in the center of a sphere (of any radius). The place where this sphere is intersected by  $\vec{P}$  exactly characterizes the information wherewith clinical practice deals. For practical execution the cartesian coordinates  $x$ ,  $y$  and  $z$  must of course be transformed to polar coordinates  $r$ ,  $\phi$  and  $\psi$  of which only the latter two are used to determine azimuth and elevation on the sphere.

Thus we can determine the position of the QRS loop of the vectorcardiogram surface in a great number of patients and deduce a diagnosis with respect to the point of intersection of  $\vec{P}$  with the sphere (the diagnosis being) obtained of course without this datum. It then appears that for specific pathological cases this point of intersection on the sphere represents an average which is characteristic for the specific disease. While other cases in this respect show no preference. The example of the atrium septum defect is very characteristic, in which the two types are distinguished by different pole vector positions. We shall go into this further on.

/109

## CHAPTER 14.

The Ventricular Gradient ( $\vec{G}$ )

In the previous chapter we showed how the pole vector  $\vec{P}$  was introduced /110 to determine the position of the surface of the vectorcardiogram in space. Another similar magnitude which also reveals a property of the loop in its entirety can be determined by the direction that the average heart vector  $\vec{H}(t)$  yields. This magnitude is designated the ventricular gradient ( $\vec{G}$ ). That  $\vec{G}$  is essentially something other than the position of the surface wherein the loop lies can be seen from figure 62, wherein the schematically drawn curves can be thought to be in a plane and therefore all have the same  $\vec{P}$  direction but different  $\vec{G}$ . (It is also possible to assume that loop 1 is in another position (thus  $\vec{P}$ ) obtained by rotation about  $\vec{G}_1$  as axis, without changing the ventricular gradient in any way.)

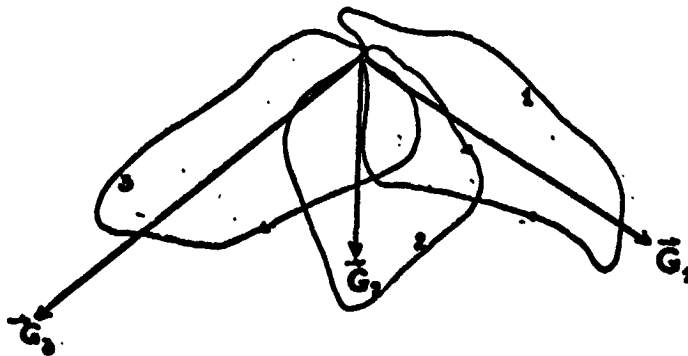


Figure 62. Three vectorcardiograms on a single plane, with different  $\vec{G}$ .

We can furnish an unequivocal formula for  $\vec{G}$ :

$$\vec{G} = \int \vec{H}(t) dt \quad (127)$$

Here we can perceive a time spread of the integral which was developed in electrocardiography, to construct  $V(t)$  (the electrocardiogram). /111

Formula (127) can be applied during the depolarization period (the QRS complex), the repolarization period (T) or both in combination. In the latter case:

$$\vec{G}_{QRS,T} = \vec{G}_{QRS} + \vec{G}_T \quad (128)$$

The action of the atria (P peak of the electrocardiogram) is not included because the integral of depolarization and repolarization yields a value for it which can be neglected.

No agreement has been reached concerning the diagnostic significance of  $\vec{G}$ . Insofar as it is of clinical value, this value derives from the property of being observable independently of the origin of excitation in the heart. As such it is an indication of the state of the myocardium.

$\vec{G}$ 's independence of the location of excitation, postulated previously, can be made plausible by a simple general hypothesis.

We assume the myocardium to be a volume of any conformation (fig. 63). At a given instant, we assume, a part of the muscle will be excited (the hatched portion in figure 63), while the rest has still an opposite polarization (see page 22). The difference between the two areas is a thin layer that runs through the heart as an (excitation) front (we will return to a discussion of the excitation front in Chapter 16).

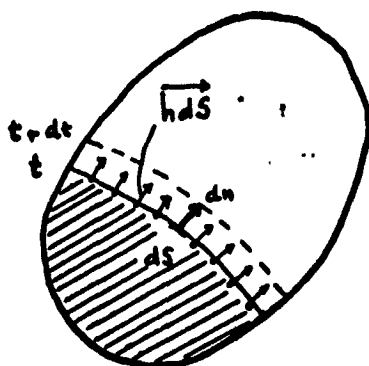


Figure 63. The excitation front of the heart at time  $t$  and  $t + dt$ . The part excited at  $t$  is hatched.

We can generally describe the front by the comparison of a surface /112  
moving in time.

$$F(x, y, z, t) = 0 \quad (129)$$

We can assume a solution for  $t$  from this expression (How this is to be done is not important for the rest of the calculation.) We then give  $t$  generally as function of  $x$ ,  $y$  and  $z$

$$t = f(x, y, z) \quad (130)$$

Generally it will be assumed that the separating surface  $F$  is exactly at the place where the source of electric heart action is seated, the "origin" therefore of the local heart vector (see Chapter 16). We thus see  $F$  as a dipole layer: indicated in the figure by arrows standing perpendicular to the layer, the dipole components  $\vec{h} dS$  which together constitute the heart vector. If we assume that each surface element component  $dS$  of  $\vec{H}$  is parallel with the surface of that element we get  $\vec{H}$  through the integral

$$\vec{H} = \int_S \vec{h} dS \quad (131)$$

wherein  $|\vec{h}|$  for the entire myocardium is assumed to be constant.

In time interval  $dt$  the front will have moved over distance  $dn$  (which does not need to be uniform everywhere) (fig. 63). On the basis of elementary analysis we can state that

$$dn = \frac{dt}{\left[ \left( \frac{\partial t}{\partial x} \right)^2 + \left( \frac{\partial t}{\partial y} \right)^2 + \left( \frac{\partial t}{\partial z} \right)^2 \right]^{\frac{1}{2}}} \quad (132)$$

or

$$dt = dn \left[ \left( \frac{\partial t}{\partial x} \right)^2 + \left( \frac{\partial t}{\partial y} \right)^2 + \left( \frac{\partial t}{\partial z} \right)^2 \right]^{\frac{1}{2}} \quad (133)$$

wherein according to (130),  $t = f(x, y, z)$ .

If we substitute this last equation in (127), we get (if for the time being we consider only the depolarization)

$$\vec{G}_{QRS} = \int_V \left[ \int_S \vec{h} dS \right] \left[ \left( \frac{\partial t}{\partial x} \right)^2 + \left( \frac{\partial t}{\partial y} \right)^2 + \left( \frac{\partial t}{\partial z} \right)^2 \right]^{\frac{1}{2}} dn \quad (134)$$

Herein  $dS dn = dv$  is a volume element of the myocardium. Therewith 113

(134) goes over into

$$\vec{G}_{QRS} = \int_V \vec{h} \left[ \left( \frac{\partial t}{\partial x} \right)^2 + \left( \frac{\partial t}{\partial y} \right)^2 + \left( \frac{\partial t}{\partial z} \right)^2 \right]^{\frac{1}{2}} dv \quad (135)$$

The expression between square brackets can now be rewritten. Thus

$$\left( \frac{\partial t}{\partial x}, \frac{\partial t}{\partial y}, \frac{\partial t}{\partial z} \right) = \overrightarrow{\text{grad } t} = \nabla t \quad (136)$$

the absolute value of the vector is accurately determined by

$$\left[ \left( \frac{\partial t}{\partial x} \right)^2 + \left( \frac{\partial t}{\partial y} \right)^2 + \left( \frac{\partial t}{\partial z} \right)^2 \right]^{\frac{1}{2}} = \left| \vec{\nabla} f \right| \quad (137)$$

If we recall that  $\vec{\nabla} f$  has the direction of  $\vec{dn}$ , the normal of the surface of excitation, and that moreover  $\vec{h}$  is directed along this normal, using  $h \equiv |\vec{h}|$  we can write for (135)

$$\vec{G}_{QRS} = \int_V \vec{h} |\vec{\nabla} f| dv = h \int_V \vec{\nabla} f dv \quad (138)$$

Some time after the excitation a recovery process (repolarization) will extend over the heart. We assume that this repolarization occurs at time  $\tau$  after depolarization, though this time interval is not the same for all parts of the myocardium. Thus  $\tau$  is a function of the coordinates:  $\tau = \tau(x, y, z)$ .

If we now wish to solve (127) for repolarization we must fill in for the time

$$t + \tau = f(x, y, z) + \tau(x, y, z) \quad (139)$$

Since the rest of the calculation proceeds similarly we mention only the result

$$\vec{G}_T = \int_T \vec{H}(t) dt = -h \int_V (\vec{\nabla} f + \vec{\nabla} \tau) dv \quad (140)$$

in which the minus sign appears because in repolarization  $\vec{h}$  has the opposite direction with respect to the area over which we now integrate. /114

According to (128) we can add (138) and (140) to determine the total

$\vec{G}_{QRS, T}$

$$\vec{G}_{QRS, T} = \vec{G}_{QRS} + \vec{G}_T = \int \vec{H}(t) dt = -h \int_V \vec{\nabla} \tau dv \quad (141)$$

It is shown hereby, with the acceptance of simple general hypotheses, that the ventricular gradient is independent of the way in which excitation passes through the heart. At the same time (141) gives meaning to the word "gradient" which previously was selected without this background. This  $\nabla \vec{V}$  results, as already noted, from the fact that the time interval between depolarization and repolarization differs from one location to another.

It is not precisely understood how this  $\nabla \vec{V}$  occurs. It has been suggested that it relates to a temperature gradient but there is no certainty about this.

If we compare the pole vector ( $\vec{P}$ ) and ventricular gradient ( $\vec{G}$ ), obviously  $\vec{G}$  can be represented geometrically in the same way as  $\vec{P}$ , i.e., we take the center of a sphere as the origin of  $\vec{G}$  and find the place where the vector intersects the sphere (thereby again shifting the data concerning the length of the side). We can thus determine two points for each loop on the sphere surface, and with this pair of points the QRS loop of the vectorcardiogram is acceptably characterized.

We can now use the point distribution on the sphere statistically, in order to obtain data concerning specific pathological conditions.

Rather than go into details we indicate the following: it is clear that magnitude  $\vec{G}$  implicitly includes time, in contrast to vector  $\vec{P}$  which depends only on the spatial position of the surface and not upon the way in which--in time-- the loop is traced. The table below shows the situation clearly:

|           | t | no t |
|-----------|---|------|
| $\vec{P}$ |   | +    |
| $\vec{G}$ | + |      |

We may ask about the possibility of determining general expressions /115 for the two magnitudes, thus filling in the blanks. It appears to be indeed possible.

We will not go further into the solution of this problem. Suffice it to say that apparently with computers it may be possible to obtain valuable diagnostic data from the results.

In this connection it should be possible to find still one more magnitude in determining the direction (a possible value) of the heart vector after a specific number ( $t$ ) of milliseconds following its "start" (for the total time scale this would approximate the explicit determination of the direction in which the loop is traced in time). This "t-millisecond vector" can again be characterized by the point of intersection with a sphere. Without doubt, on condition that the value of  $t$  is suitably selected, the distribution of the points on the sphere would yield valuable information in distinguishing specific types of deviations. For the frontal projection alone experiments have already been performed (a circle replaces the sphere) and not without success. Attacking the problem of the 3-dimensional case is decidedly worth the trouble.

## CHAPTER 15.

### The Multipole Effect

In our consideration of the heart vector we have so far dealt exclusively with the dipole effect, ignoring the components of higher poles. Only in passing (page 27) we indicated that the potential at a point P caused by a cluster of point sources in a homogeneous medium can be described as the sum of the single pole, dipole, quadrupole, etc. effect. /116

$$V_P = \frac{\phi_1}{r} + \frac{\phi_2}{r^2} + \frac{\phi_3}{r^3} + \dots \quad (22)$$

One of the requirements for rapid convergence of the series, that the first term be zero, made us set the potential as equal to the second term. In this chapter however we shall go somewhat more deeply into the general theory of potentials, in connection with Chapter 5, to be able to examine the way in which it is expressed in terms of components of higher poles, especially the quadrupole.

We first consider development (22) in more detail. We consider a plurality of point sources  $I_1$  with a small region of a homogeneous infinitely extended medium. The distance of each source from the origin O of a set of coordinates, provisionally random, among the point sources, we designate  $r_1$  (fig. 64). The potential  $V_P$  at a point P, distance  $r$  from O, becomes distance  $R_1$  from  $I_1$  because we also assume  $\frac{I}{4\pi\epsilon} = q_1$ , given by:

$$V_P = \sum_1 \frac{q_1}{R_1} \quad (142)$$

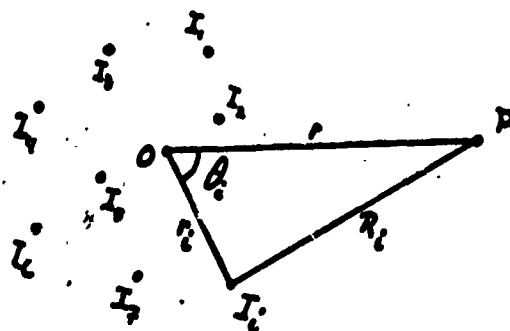


Figure 64. Computation of point P.

We shall convert this expression. By the cosine rule:

$$R_i^2 = r^2 + r_i^2 - 2rr_i \cos \theta_i \quad (143)$$

whereby for  $V_P$  we have:

117

$$V_P = \sum_i \frac{q_i}{(r^2 + r_i^2 - 2rr_i \cos \theta_i)^{\frac{1}{2}}}$$

or

$$V_P = \sum_i \frac{q_i}{r} \left[ 1 + \left( \frac{r_i}{r} \right)^2 - 2 \frac{r_i}{r} \cos \theta_i \right]^{-\frac{1}{2}} \quad (144)$$

Although it is "generally" known that an expression in Legendre polynomials can be used for (144) we will go into the matter briefly. Formula (144) is in the form

$$P(x, y) = (1 + y^2 - 2xy)^{-\frac{1}{2}} \quad (145)$$

This function can be developed in a series, as we shall briefly note. If  $F(x, y)$  is differentiated  $n$  times according to  $y$  at point  $y = 0$ , the result is:

$$\left( \frac{d^n F(x, y)}{dy^n} \right)_{y=0} = 2^n P_n(x) \quad (146)$$

where  $P_l(x)$  are Legendre polynomials

$$P_0 = 1 ; P_1 = x ; P_2 = \frac{1}{2}(3x^2 - 1) ; P_3 = \frac{1}{2}(5x^3 - 3x) ; \dots$$

If we develop  $F(x,y)$  at  $y = 0$ , we get

$$F(x,y) = F(x,0) + y \left( \frac{\partial F}{\partial y} \right)_{y=0} + \frac{y^2}{2!} \left( \frac{\partial^2 F}{\partial y^2} \right)_{y=0} + \dots + \frac{y^l}{l!} \left( \frac{\partial^l F}{\partial y^l} \right)_{y=0} \quad (147)$$

Applying (146), this is written as:

$$F(x,y) = (1 + y^2 - 2xy)^{-\frac{1}{2}} = \sum_l y^l P_l(x) \quad (148)$$

We can only apply this expression as the right hand member converges, which will be the case when  $y < 1$  and  $P_l(x) \leq 1$ . We can assume that the latter is true when  $|x| \leq 1$ . /118

The general expression (148) is adapted to the equation of potentials (144). We see that:

$$V_P = \sum_l \frac{q_l}{r} \sum_l \left( \frac{r_l}{r} \right)^l P_l(\cos \theta_1) \quad (149)$$

or

$$V_P = \sum_l \frac{\sum_l q_l r_l^l}{r^{l+1}} P_l(\cos \theta_1) \quad (150)$$

When point P is far enough outside the clustered sources,  $\frac{r_l}{r} < 1$ , so that conditions for convergence of the series are fulfilled. If we now take:

$$\sum_l q_l r_l^l P_l(\cos \theta_1) = Q_l \quad (151)$$

we have a simple expression for the multipole development:

$$V_P = \sum_l \frac{Q_l}{r^{l+1}} \quad (152)$$

wherein  $Q_l$  is independent of  $r$ . In our case, since as much electricity flows out from every volume element of the heart as comes in, the term for the single pole is discarded, so that:

$$Q_0 = \sum_l Q_l = \frac{1}{4\pi\sigma} \sum_l I_l = 0$$

We shall now clarify this theory by some examples.

1. We consider a current doublet. The origin of the system of coordinates is chosen in the middle, between the + and - poles. We calculate the potential at a point P and distance  $r$  from O (fig. 65) by means of (152) and (151). If the data of figure 65 are substituted in this formula, taking into account of the current by sign, and recalling that the angle between  $r$  and  $r_-$  is 119 equal to  $\theta + \pi$ , and with consideration that  $|r_+| = |r_-| = U$ , it is easy to understand that:

$$V_P = \frac{2I}{4\pi\sigma} \left( \frac{U P_1(\cos \theta)}{r^2} + \frac{U^3 P_3(\cos \theta)}{r^4} + \dots \right) \quad (153)$$

The first term is the--anticipated--dipole component, and the second term represents the octapole effect. We see that the quadrupole term is absent in this selection.

2. As second example, we select a so-called dipole shell. This is a circular disk (radius  $a$ ) whose midpoint coincides with the origin of our system of coordinates and which has a balanced dipole distribution so that the right side of the disk contains the positive pole and the left side of it the negative (fig. 66) (a special case of a so-called "dipole disk") (see also Chapter 16). We again calculate the potential at point P, distance  $r$  from O. We limit

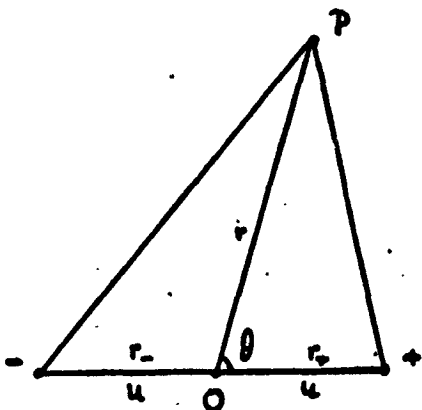


Figure 65. A current doublet with origin of the system of coordinates halfway between the poles.

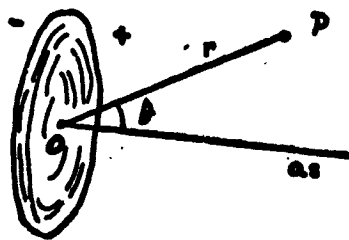


Figure 66. A homogeneous dipole disk.

ourselves to the case  $r > a$ . For reasons of symmetry  $V_P$  is the function of  $r$  and  $\theta$ . Since this function--which we again write as a series--must be finite for  $r \rightarrow \infty$ , the series will contain only negative powers of  $r$ . Since  $\frac{P_l(\cos \theta)}{r^{l+1}}$  satisfies the Laplace equation,  $\nabla^2 V = 0$ , in analogy to (152) for  $V_P$ :

$$V_P = b_1 \frac{P_1(\cos \theta)}{r^2} + b_2 \frac{P_2(\cos \theta)}{r^3} + b_3 \frac{P_3(\cos \theta)}{r^4} + \dots \quad (154)$$

Constants  $b_l$  here are to be determined by calculating the potential for 120 the special case of a point on the axis. Therefore  $\theta = 0$  so that since  $P_l(\cos 0) = P_l(\cos 1) = 1$ :

$$V_{as} = \frac{b_1}{r^2} + \frac{b_2}{r^3} + \frac{b_3}{r^4} + \dots \quad (155)$$

$V_{axis}$  is to be calculated by integration of

$$V \, dS = \frac{dD \cos \varphi}{4 \pi s^2} \, dS \quad (156)$$

on the dipole shell. Here  $D$  is the total dipole moment of the disk,  $dD$  thus is the dipole density per area thereof,  $s$  is the distance of a surface element  $dS$  to point  $Q$  on the axis, distance  $r$  from  $O$  and  $\varphi$  the angle as defined earlier (see fig. 67) (fig. 16 and (21) page 26): since  $\theta$  of figure 16 is other than that of figure 66 we have written  $\varphi$  in (156)). When we make the surface element annular:

$$dD \, dS = \frac{D}{\pi a^2} \cdot 2\pi \rho \, d\rho \quad \text{whereas}$$

$$s = (\rho^2 + r^2)^{\frac{1}{2}}. \quad \text{Thereby } V_{\text{axis}} \text{ becomes}$$

$$V_{\text{as}} = \int_{\rho=0}^a \frac{D}{\pi a^2} 2\pi \rho \, d\rho \cdot \frac{1}{4\pi(\rho^2 + r^2)} \cdot \frac{r}{(\rho^2 + r^2)^{\frac{1}{2}}} \quad (157)$$

Calculation of the integral yields

$$V_{\text{as}} = \frac{Dr}{4\pi a^2} 2 \left[ r^{-1} - (a^2 + r^2)^{-\frac{1}{2}} \right] \quad (158)$$

To compare this expression with (155) we write it as a series.

$$V_{\text{as}} = \frac{D}{4\pi a^2} \left( \frac{a^2}{r^2} - \frac{3}{2 \cdot 2!} \frac{a^4}{r^4} + \frac{3 \cdot 5}{2^2 \cdot 3!} \frac{a^6}{r^6} - \dots \right) \quad (159)$$

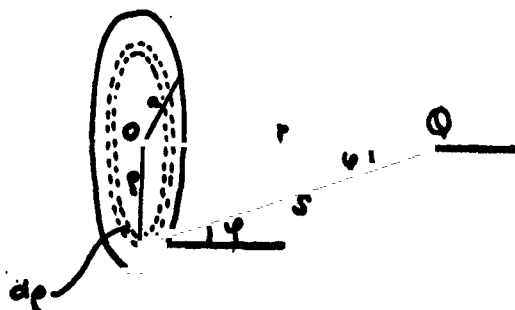


Figure 67. In the calculation of the potential at a point  $Q$  and distance  $r$  from a homogeneous dipole shell.

Using (155) we determine from (154) that:

[121]

$$V_P = \frac{D}{4\pi r^2} \left[ P_1(\cos \theta) - \frac{1}{2} \frac{a^2}{r^2} P_3(\cos \theta) + \frac{5}{8} \frac{a^4}{r^4} P_5(\cos \theta) - \dots \right] \quad (160)$$

The first term is again the dipole effect, the second the octapole component, etc. It is striking that the quadrupole, like all the righthand terms with un-even powers of  $r$ , is again missing.

3. In the following example the origin of the system of coordinates is place at any point on the connecting line between two current poles  $I$  and  $-I$ , combined to form a current doublet (fig. 68) (in the limit  $r_1 - r_2 \rightarrow 0$  the

doublet again goes over into a dipole). It is again readily seen (see example 1) that the potential at a point  $P$  at distance  $r$  from  $O$  as given (for the meaning of  $r_1$  and  $r_2$ , see figure 68):

$$V_P = \frac{I}{4\pi r} \left[ \frac{r_1 - r_2}{r^2} P_1(\cos \theta) + \frac{r_1^2 - r_2^2}{r^3} P_2(\cos \theta) + \frac{r_1^3 - r_2^3}{r^4} P_3(\cos \theta) + \dots \right] \quad (161)$$

This time the quadrupole term is present. In view of the fact that this was not the case in the first example, and since we did not use a different position of  $P$  with reference to the doublet, the presence of the quadrupole term must be explained by the selection of the coordinates. In general we may say that the shift in position of the origin has no effect on the dipole, or on higher poles.

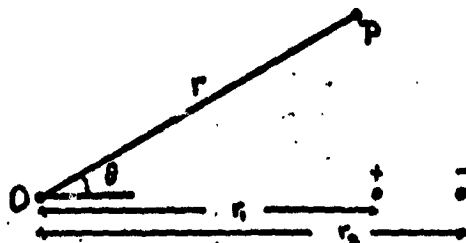


Figure 68. The origin at any point on the axis of a current doublet.

4. Finally we take not one but rather two different current doub- [122]

lets. For the sake of simplicity we assume that it is true of all poles that a current  $I$  is introduced and carried off and that the difference in the "doublet density" will be determined by the mutual distance of the poles. Somewhere on the line on which the poles are assumed to lie (fig. 69) we select the origin. The potential at  $P$  is again calculated by application of (151) and (152):

$$V_P = \frac{I}{4\pi\epsilon} \left[ \frac{P_1(\cos\theta)}{r^3} (r_1 - r_2 - r_3 + r_4) + \frac{P_2(\cos\theta)}{r^5} (r_1^2 - r_2^2 + r_3^2 - r_4^2) + \dots \right] \quad (162)$$

The presence of the quadruple term is now not only a consequence of the choice of the origin but also of the fact that here we have an intrinsic quadrupole.

We can now ask ourselves how far it is possible to make the quadrupole component minimal by an appropriate choice of the origin of the system of coordinates. It may be possible in vectorcardiography to get around the difficulties that result from the inexact choice of the dipole approximation, as we indicated in earlier chapters. It will be immediately understood that it is possible to effect this. If we shift the origin by a distance  $\delta$  we can write for  $r_1^2 - r_2^2$  in the quadrupole term in (161):  $(r_1 + \delta)^2 - (r_2 + \delta)^2$ . If the quadrupole is to disappear, then:

$$(r_1 + \delta)^2 - (r_2 + \delta)^2 = 0 \quad (163)$$

From this condition it follows for  $\delta$  that:

$$\delta = -\frac{1}{2} (r_1 + r_2) \quad (164)$$

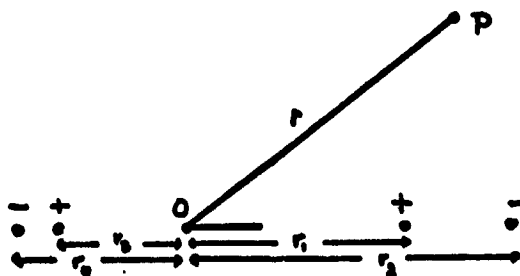


Figure 69. Two different current doublets.

In reality, however, the origin can be so far removed from the /123/  
heart, even from the human body altogether, that violence is done to physical  
reality, while the intrinsic quadrupole can never be eliminated. Although the  
~~medical~~ side may well suggest experiments in this direction, we believe that no  
meaningful practical outcome is to be anticipated from them.

It would be better to assign the coordinates system its origin in  
the "midpoint" of the heart, in order experimentally to determine the values  
that characterize the quadrupole effect.

Before we go into this more deeply, we turn back to theory.

We saw in (152) how the potential at a point P, caused by a cluster of  
sources, could be expressed by a series in negative powers of r. We shall  
write this series in a somewhat different form for point P (fig. 64) by using  
polar rather than cartesian coordinates. We take  $Q_1$  from another formula (142)  
and it becomes the function of azimuth and elevation of the radius vector at  
P.

Instead of  $Q_1$ --without going into the explicit form of it--we now  
write  $\phi_n$  (where  $n = l + 1$ , to make the index of  $\phi$  agree with the power of  
r). Thus

$$V_P = \sum_{n=1} \frac{\phi_n}{r^n} = \frac{\phi_1}{r} + \frac{\phi_2}{r^2} + \frac{\phi_3}{r^3} + \dots \quad (22)$$

We have thus recovered the first statement of the series (see (22) page 116  
and page 27).

There appears to be a practical advantage when we introduce still another  
formula, combination of cartesian and polar coordinates. This can be done by  
means of (22). We also return to the single dipole (page 26). We saw there  
that for the potential at a point P at distance r from the dipole.

$$V_P (:) = \frac{\cos \theta}{r^2} \quad (165)$$

If we multiply numerator and denominator by  $r$

$$V_P (:) = \frac{r \cos \theta}{r^3} \quad (166)$$

we can see the (cartesian)  $x$  coordinate in  $r \cos \theta$ . Since  $\theta_1$  as we observed, is a function of the direction cosine at  $P$ , we can attempt to generalize it by writing the term for the dipole of (22):

$$\frac{\phi_2}{r^2} = \frac{r \phi_2}{r^3} \quad (167)$$

and considering whether it is generally possible to substitute

$$V_{P(\text{dipool})} = \frac{r \phi_2}{r^3} = \frac{ax + by + cz}{r^3} \quad (168)$$

wherein  $x$ ,  $y$  and  $z$  are the customary cartesian coordinates and  $a$ ,  $b$  and  $c$  are determined by the dipole density. It is easily proved that (168) indeed satisfies the Laplace equation:  $\nabla^2 V = 0$ .

A similar operation applied to the quadrupole term entails investigation of the permissibility of writing

$$V_{P(\text{quadrupool})} = \frac{r^2 \phi_3}{r^5} = \frac{kx^2 + ly^2 + mz^2 + syz + txz + uxy}{r^5} \quad (169)$$

It appears that this expression satisfies Laplace, though upon condition that

$$k + l + m = 0 \quad (170)$$

Similar expressions can also be found for higher poles, again under certain conditions which satisfy  $\nabla^2 V = 0$ . We shall not go any farther into this.

We see by these simple ways of formulating--simple, because complex generalized Legendre functions are rare--that the dipole, as we anticipated, is given by three magnitudes which correspond to the three vector components of the dipole moment. The quadrupole term includes six magnitudes, k, l, m, s, t and u. However, the extra relation between the first three makes it so /125 that the quadrupole field is characterized not by six magnitudes but rather by  $6 - 1 = 5$ .

We speak here of a tensor (here a tensor of the second order, because we designate a tensor of the first order as a vector. A vector and a tensor are distinguished by the way in which they are transformed by rotation of the system of coordinates. The components of a vector transformed by rotation are linear functions of the direction cosines which determine the rotation, while in the same case a tensor of the second order is quadratically transformed).

From the above we see that if we are not satisfied in "vector" cardiography with the vectoral dipole effect but also take the quadrupole component into account, we must solve for five unknowns all at one time! But then the previous minimum of four electrodes is no longer sufficient. The  $3 + 5 = 8$  independent values which we now have to determine require 8 independent differences of

potential, hence at least 9 electrodes (by which we do not mean to say that any number of electrodes greater than four would not be meaningful in "classic" vectorcardiography. The contrary is true, as appears from Chapter 11).

We now again follow the procedure that we applied earlier (page 40). We write the difference of potential between a certain electrode somewhere on the body surface and a reference electrode (which of course must also be located

(on the body surface) as a linear combination of the 3 dipole and 5 quadrupole components

$$V_{P_1} - V_{P_0} = V_1 = a_1 X + b_1 Y + c_1 Z + d_1 Q_1 + e_1 Q_2 + f_1 Q_3 + g_1 Q_4 + h_1 Q_5 \quad (171)$$

The 8 coefficients  $a_1 \dots h_1$  can be found by model experiments, analogous to those described earlier (chapter 8) by the "lead vectors." We now find not only a lead vector ( $\overrightarrow{abc}$ ) but at the same time a lead tensor ( $\overrightarrow{defgh}$ ).

If we do this for 8 different pairs of electrodes

/126

$$V_{P_1} - V_{P_0} = V_1 = a_1 X + b_1 Y + c_1 Z + d_1 Q_1 + e_1 Q_2 + f_1 Q_3 + g_1 Q_4 + h_1 Q_5$$

$$V_{P_2} - V_{P_0} = V_2 =$$

$$V_{P_8} - V_{P_0} = V_8 = a_8 X + b_8 Y + c_8 Z + d_8 Q_1 + e_8 Q_2 + f_8 Q_3 + g_8 Q_4 + h_8 Q_5 \quad (172)$$

and determine 64 constants  $a_1 \dots h_8$  on the model, then conversely we can find the 8 components which describe the electric activity of the heart (see page 40), using differences of potential measured on the human body.

Instead of 8 voltages, we can select 9, 10, ... . The problem, which is now overdetermined, can be solved by application of the method of least squares (and an electronic computer) as octapoles and higher poles are neglected.

Naturally it is not possible to join the five quadrupole components in a single figure, as the vectorcardiogram brings the behavior of the dipole vector at the time of the heart cycle into the picture. We can do no more than record the Q's as five separate time functions and assume the combination of

$$X = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \alpha_4 v_4 + \alpha_5 v_5 + \alpha_6 v_6 + \alpha_7 v_7 + \alpha_8 v_8$$

$$Y = \beta_1 v_1 +$$

$$Z = \gamma_1 v_1 +$$

$$Q_1 = \delta_1 v_1 +$$

$$Q_2 = \epsilon_1 v_1 +$$

$$Q_3 = \zeta_1 v_1 +$$

$$Q_4 = \eta_1 v_1 +$$

$$Q_5 = \nu_1 v_1 + \nu_2 v_2 + \nu_3 v_3 + \nu_4 v_4 + \nu_5 v_5 + \nu_6 v_6 + \nu_7 v_7 + \nu_8 v_8$$

(173)

these curves in the consideration that lead to diagnosis of the heart /127

which is being examined.

The method here expounded, however simple it may seem, is so complex and hence so uncertain in experimental results that clinical adaptations are still exceptional.

## CHAPTER 16

## The Origin of the Electric Activity of the Heart

In all that has been said so far concerning the electric activity of 128 the heart, we have bypassed the phenomena that may be considered responsible. It is the purpose of this chapter to go into the matter. It should be emphasized that no exhaustive treatment is contemplated. We content ourselves with a rough outline in this paper, so that the basis of our study will not be forgotten.

We have seen that the representation of the electric activity of the heart using a single dipole or a constant location yields no truly satisfactory result. In Chapters 14 and 15 we applied another way of looking at it which appears more real, by introduction of the excitation front (dipole front or dipole shell). This view is developed by measurements of potential at the surface of the heart. Figure 70 shows how the excitation front can be represented.

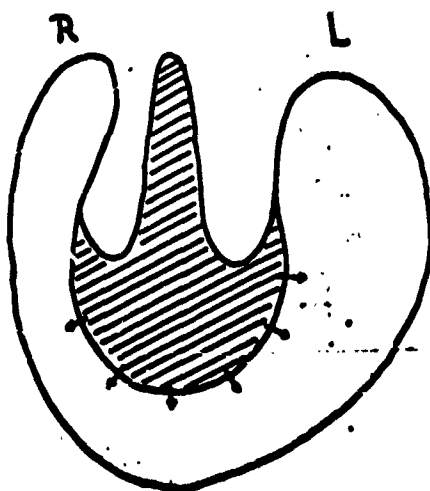


Figure 70. Cross section of the heart with excitation front.

We show a cross section of the heart, one part of which is excited (the hatched portion) while the rest conversely is at rest. The electric dipole effect is directed from the excited part to the non-excited part.

More refined measuring techniques of recent years, whereby measurements have been made in the myocardium, have confirmed and extended hypotheses concerning the excitation front. As preface to the results of these experiments, we will describe certain features characterizing the method used. /129

To penetrate into the heart, a probe is needed, furnished with electrodes. A fine hollow needle has been used, which has small apertures (0.1 mm diameter) over part of its length. These openings which are spaced at distances on the order of one millimeter contain the outer ends of wires which are bundled in the needle: thus in fact a system of microelectrodes between each pair of which the voltage can be measured. If one or more of these needles is thrust into the heart (a 2-unit system has been applied), it is possible by measuring the voltages between the electrodes as a function of time to get an impression of the properties of the excitation front.

Use can be made of the bipolar method or the unipolar. In the latter case the voltage is measured between the needle electrodes and a fixed electrode placed somewhere on the body, for example, while in the former case voltages between adjacent pairs of electrodes of the needle are measured. The course of the potential as it appears in the two respective systems when the excitation front passes an electrode is schematically shown in figure 71.

From the distance separating the electrodes in the needle, the difference in time between the same phenomena in the successive electrodes and the form of the curves, the speed with which the front is propagated and the thickness of the layer are measured.

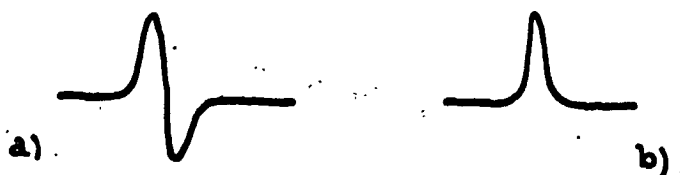


Figure 71. a, unipolar characteristic; b, bipolar characteristic.

It appears that the thickness of the front must be a maximum of 1 mm. With respect to the measurement of the myocardium, this is so little that we can speak of a true dipole shell.

For speed of propagation different values have been obtained in measurement of different kinds of tissue:

|                                       |                   |             |
|---------------------------------------|-------------------|-------------|
| Myocardium (heart muscle)             | 30 to 50 cm/sec   | <u>/130</u> |
| Purkinje (stimulus conducting) system | 100 to 170 cm/sec |             |
| Atrioventricular node                 | 5 to 12 cm/sec    |             |

The first two values relate to the QRS complex. Hence for the contraction phase of the ventricles (the electric phenomena occurring in the recovery process appear to be less readily measureable). The results agree with what we should anticipate from the electrocardiogram, where the QRS complex is accomplished in  $\pm 60$  ms. (If the heart measurement is set at an order of 8 cm and propagation speed is selected as 140 cm/sec, then for the duration of the QRS complex  $8/140 = 60$  ms).

We address special attention to the low speed at which excitation is propagated in the tissue of the atrioventricular node. We find here the explanation of the delay occurring between stimulation of the atria and the ventricles which makes the latter contract after the first have completed their contraction, thus making efficient ejection possible.

How can we conceive the dipole surface from the microstructure of the myocardium?

Like other muscle, the myocardium consists of fibers which vary in thickness from 5 to 100  $\mu\text{m}$  (in chapter 2 the anatomy of the heart is described in more detail). Each of these fibers can be considered as a small cylinder with a semipermeable membrane 70 to 80  $\text{\AA}$  thick for a wall, said membrane having very special properties. An old image that has not lost its meaning describes the fiber as a cylindrical capacity with the membrane as insulator. In the polarized state it is negative on the inside and positively charged on the outside (fig. 72), thus giving rise to a specific membrane voltage. Under specific conditions there is a "breakdown" place somewhere, so that the membrane becomes conductive. The capacitor then discharges (fig. 73) so that the difference is levelled. The membrane now has the special property that it becomes conducting--"breakdown"--when the voltage between the inner surface and the outer surface of the fiber drops below a certain value. (This is in contrast to the behavior of an ordinary capacitor which breaks down when the voltage rises above a limit value.) This phenomenon makes it so that when the excitation process has an opportunity to start somewhere, the disturbance of

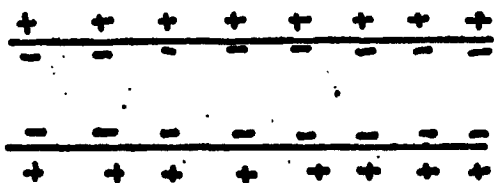


Figure 72. Muscle fiber in polarized condition as "charged capacitor."

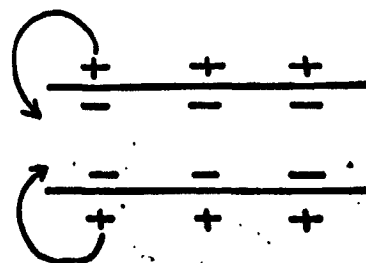


Figure 73. At a breakdown point the "capacitor" can discharge.

equilibrium is propagated along the whole fiber, whereby occasion is furnished for an electric phenomenon.

We shall discuss some details of this roughly sketched outline. We must penetrate again to the innermost part of our material. The most common experimental method will be examined as above, before we report the results of the experiments.

To penetrate inside a muscle fiber, we must have a needle electrode of uttermost thickness. This has been found in a drawn glass tube whose maximum diameter can be reduced to  $\pm 1 \mu\text{m}$ --open at both sides and filled with an electrolyte, e.g., KCl. To prevent unmanageable resistances the tube is widened immediately beyond the tip (fig. 74). The total resistance of such an electrode is on the order of  $10 \text{ M}\Omega$  so that high input impedance requirements of the measuring apparatus can be established. Fortunately the voltages to be measured are not too small:  $\pm 0.1$  volt.

Let us first consider the resting state, e.g., of the ventricle--that is to say, the situation in which the myocardium is polarized. How then does the capacitor become charged? It is known that the  $\text{K}^+$  ions play an important part in the buildup of membrane potential. It has been found that the  $\text{K}^+$

/132

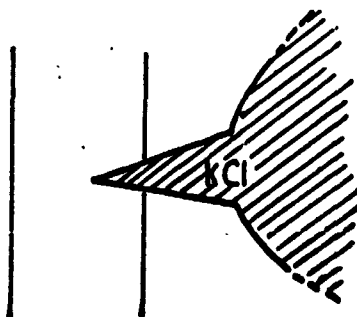


Figure 74. Glass electrode inserted in a muscle fiber.

concentration inside the fiber is many times higher than outside. If we proceed in theory from an electrically neutral state (i.e., same potential on both sides of the membrane) then, because the  $K^+$  ions pass through the fiber wall and the other ions cannot, or are only barely able to do so, a diffusion of  $K$  ions to the outside occurs, because the potassium concentration there is minimal. Consequently there is a surplus of anions remaining in the fiber, which causes the development of an electric field through which the potassium ions will be drawn back until the diffusion is exactly compensated, i.e., until a stable condition is reached. If we select an  $x$  coordinate perpendicular to the membrane which is positive at the inside, we can describe the potassium transport to the outside, applying the law of diffusion

$$m_K = -D \frac{dC}{dx} \quad (174)$$

in which  $m_K$  is the quantity of  $K^+$  ions per time unit and per surface unit,  $D$  is the diffusion constant and  $\frac{dC}{dx}$  is the drop of concentration in the membrane.

From physical chemistry it follows that the absolute value of potassium transport in the opposite direction is

$$m_K = \beta \cdot e \cdot C \frac{dV}{dx} \quad (175)$$

where  $\beta$  is the motility of the ions and  $e$  is the elementary charge. In the stable condition, of course

$$-D \frac{dC}{dx} = \beta \cdot e \cdot C \frac{dV}{dx} \quad (176)$$

By integration of this differential equation, it follows that for the difference of potential between the inside and the outside of the membrane

$$\Delta V = \frac{D}{\rho e} \ln \frac{C_1}{C_2} \quad (177)$$

where  $C_1$  is the potassium concentration inside and  $C_2$  the concentration outside the cylinder. /133

We see from (177) that the membrane voltage is proportional to the log of the ratio of concentration. If we examine this ratio in various types of animals, it appears that the differences are such that no difference of membrane potentials greater than a factor of 3 is to be anticipated. Such a difference has been measured.

We now consider what happens in excitation. As already observed in excitation, the membrane (the fiber wall) suddenly becomes conductive and a breakdown takes place in it which is propagated from its point of origin along the whole fiber. It is obvious to assume that at the time of discharge the voltage in the "capacitor" is zero. This seems to be incorrect: the voltage changes sign at about 1/4 of its original value (whereby the interior becomes positive, the outside negative) as seen in figure 75, where the voltage over the membrane at a specific location is represented as a function of time.

The dashed line in the figure is the zero line.

The form of the potential curve is specific for the type of fiber, i.e., for myocardium, Purkinje system and atrioventricular node there are different

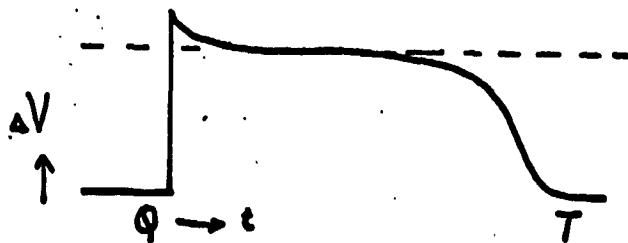


Figure 75. Voltage characteristic as measured at a specific location in the muscle as a function of time.

curves. We shall not consider the entire course of the curve in detail but limit ourselves to the sudden potential jump at its inception.

How will the electric field appear as a result of excitation? The dipole layer is broken open as a result of the breakdown. Let us now examine a fiber on the breakdown side where rest conditions still prevail (fig. 76a). We can then think of the opening as filled with two layers, each with opposite charge (fig. 76b).

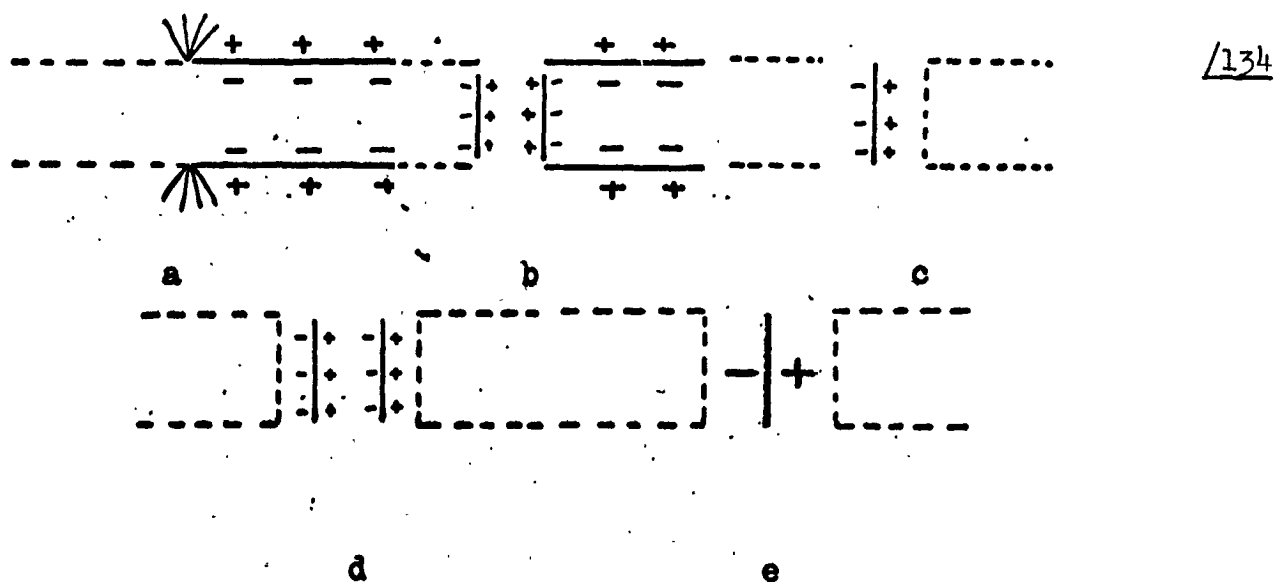


Figure 76. The distribution of charge in an active muscle fiber <sup>leads</sup> to the dipole, the origin of the heart vector.

The inner layer completes as it were the membrane of a closed dipole layer. The field at the outside consequently must be zero, the second layer, only provided to keep the total charge unaltered, then representing a fictive dipole layer (fig. 76c). A similar reasoning applied to the excited part of the fiber (with a reversed charge distribution) also yields a dipole layer (fig. 76d). We now let these two like-directed dipole layers fuse so that we have a single dipole layer over the whole fiber cross section (fig. 76e).

If we recall that the muscle fibers lie next to each other in bundles and that they are almost simultaneously stimulated by the Purkinje system, we can conceive how the excitation front, hence the dipole layer, comes into existence and moves through the heart at the time of a heart beat.

For the sake of simplicity of the account, we do not consider in the /135 above the part played by the sodium ions. Also certain phenomena are left partly unexplained. We felt that a treatment of them lay beyond the scope of the present paper.

## CHAPTER 17.

### Clinical Application of Vectorcardiography

(by G. van Herpen)

#### 1. General

/136

The myocardium comprises a network of muscle fibers, a network in which distinct preferred <sup>e</sup>directions are to be observed, so that in separate regions the fibers run distinctly parallel to each other. The preferred direction is thus not the same in all locations and in all orientations: the heart has a definite "fiber architecture." A single <sup>muscle</sup> fiber is a column of cells surrounded by a common sarcolemma. The fiber comprises a great number of contractile fibrils.

A normal muscle fiber presents a membrane potential. Between the positive charge at the outside and the negative on the inside a difference of potential prevails which is approximately 80 millivolts.

When the fiber is excited this charge difference is partly lost and the difference of potential drops to e.g. 40 millivolts. The excitation is propagated over the fiber as a disturbance of the membrane potential. Where the depolarization has occurred, the membrane has less positive charge on the outside than before the depolarization. There is likewise a difference of potential between the membrane surface ahead of and behind the excitation front.

"Ahead" is positive with reference to "behind" ( $\pm 20$  mV). Since the fibers as mentioned are substantially parallel to each other they will draw a common excitation front over the fiber, since they are stimulated at least simultaneously and in the same perpendicular plane.

This front can therefore be considered as a dipole surface. The conformation and direction of the front and its progress are determined by the fiber orientation and the moment of stimulation of the fiber system. In addition to ordinary muscle fibers the heart possesses a specific stimulus conduction system that has become differentiated from muscle fiber. The stimulus (excitation, depolarization) moves through the fibers of this system much more rapidly ( $\pm 10x$ ) than in ordinary muscle fibers. Finally the stimulus arrives in the region of the myocardium for which it is specific, and it is divided onto the muscle fibers to spread in the fiber network at a reduced speed. /137

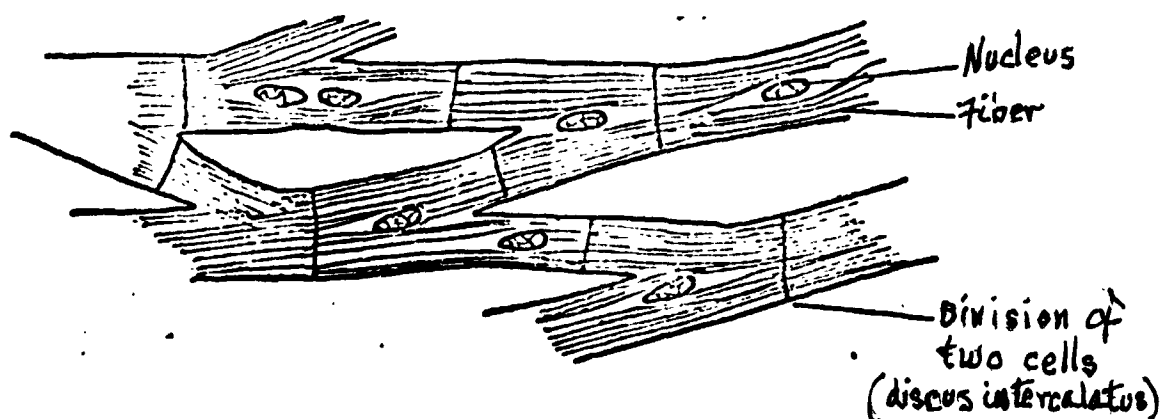


Figure 77a. Myocardial fiber with branching.

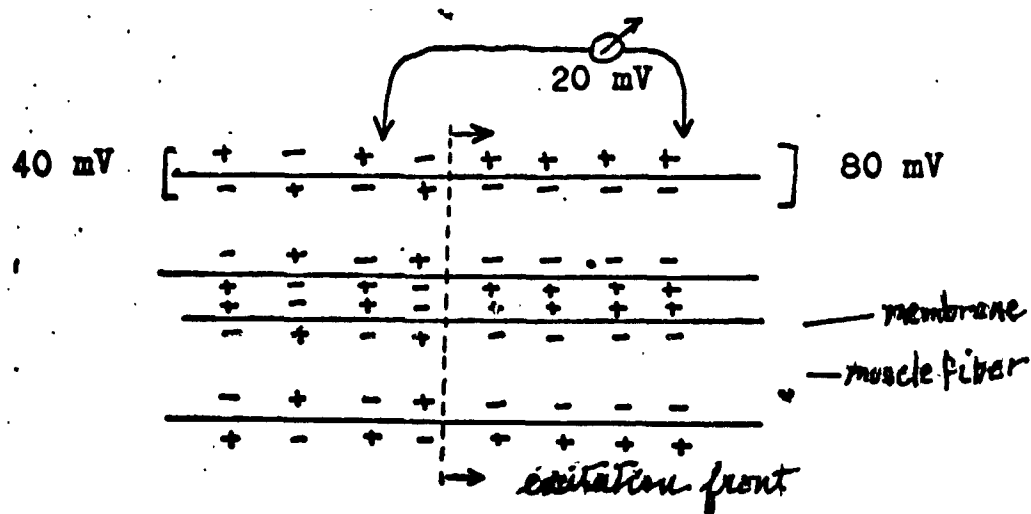


Figure 77b. Membrane potential and depolarization.

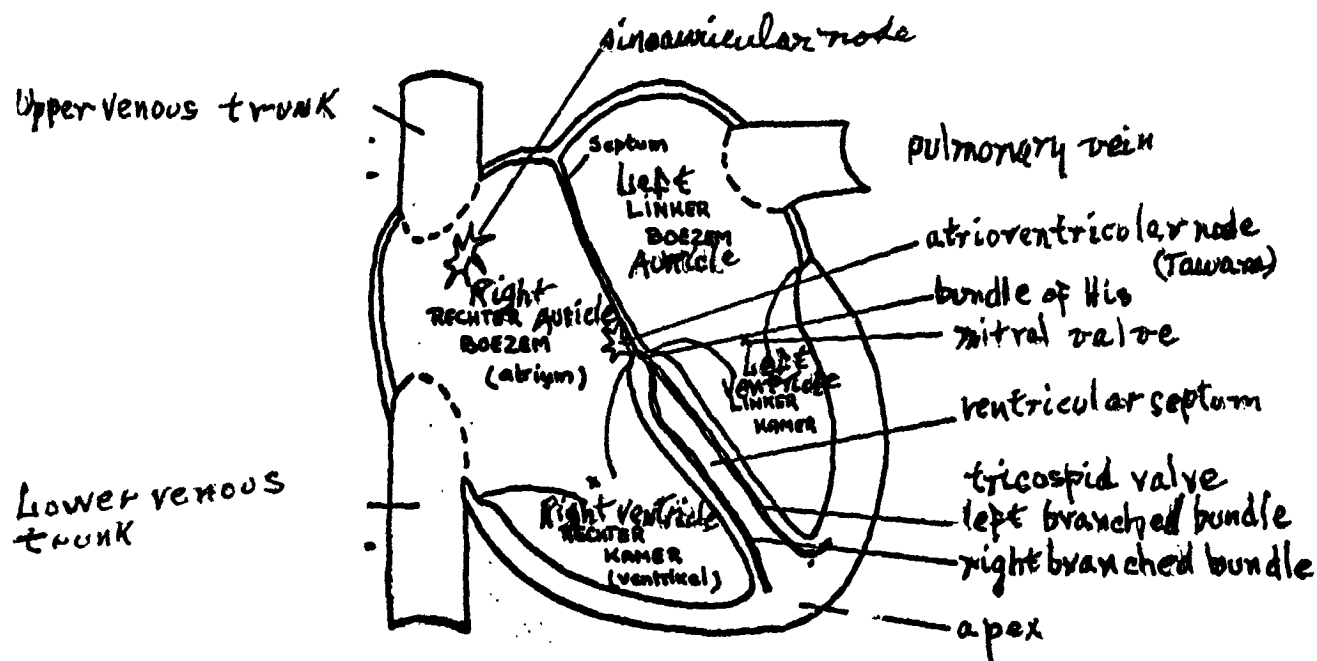


Figure 78. Heart with stimulus conduction system.

In this way it is possible that the excitation (and therewith the contraction) of the myocardium begins in different regions almost simultaneously and proceeds in an ordered manner.

The stimulus arises in the sinoauricular node and spreads over the atria without assistance on the part of a specialized conducting system. It reaches the atrioventricular node, whence it is directed to the ventricles by the specific conducting system (bundle and branched bundles). /138

In broad outline we can say that the excitation of the muscle first takes place in the septum. Subsequently the endocardial surface of the ventricle walls is excited via branches of the stimulus conducting system. The epicardial surface follows later through muscle conduction. The spread is centrifugal from the apex.

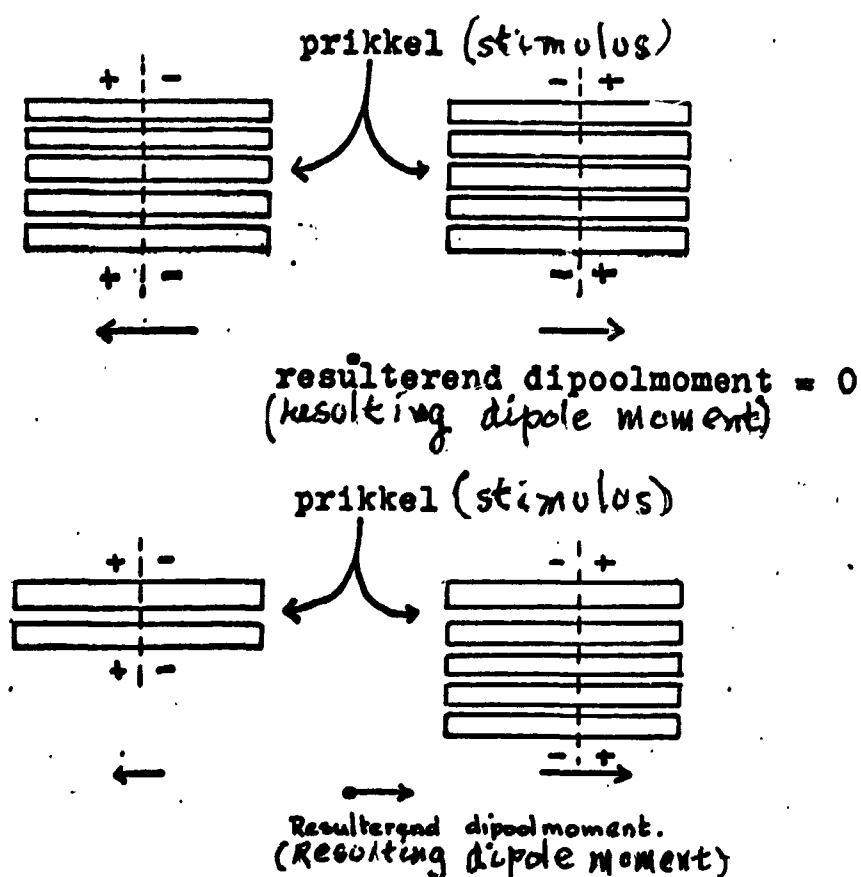


Figure 79. Effect of direction of the depolarization front and of muscle mass on the resulting dipole moment.

In this way the moving dipole front varies <sup>in</sup> place and time, causing in turn the "heart vector" (the resulting dipole moment). It is clear that the manner of spread via the specific stimulating system has a strong determining effect on the behavior of the vector. But peripheral conduction through the muscle itself is of great significance also. When the thickness of the muscle wall is greater locally, the spread takes longer and this affects the direction of the vector. When the wall is thicker the size of the vector can also be increased because the fibers of the wall will be excited simultaneously. On the other hand it can happen that parts of the front, when they move in opposite directions, can mutually compensate each other's effect. If the muscle mass concerned is not uniform there is an external effect.

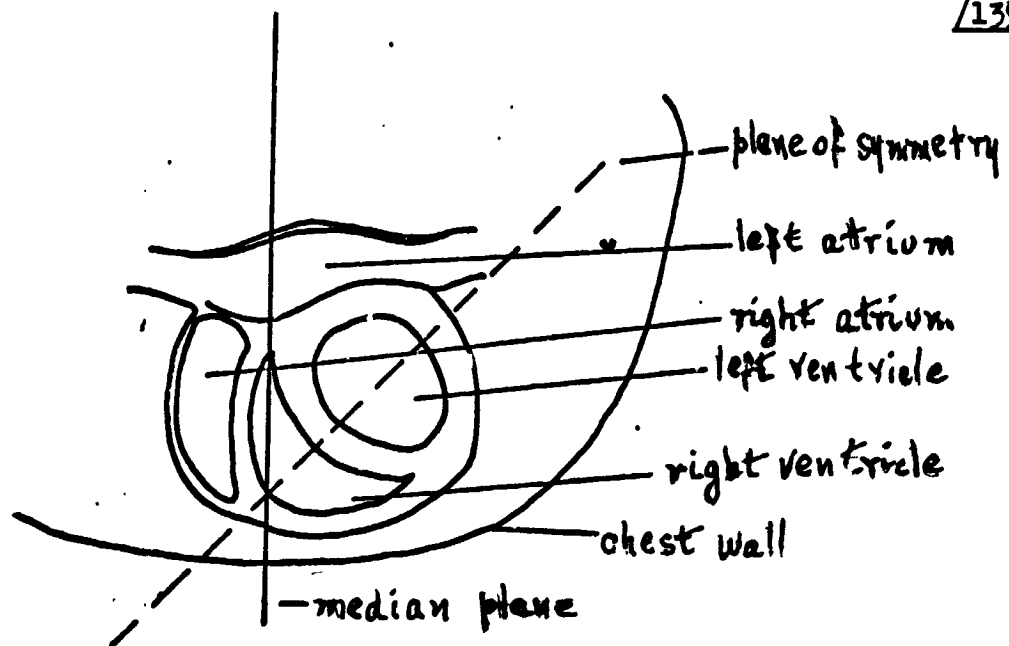


Figure 80. Transverse section of the heart.

If the myocardium were a sphere with uniform wall thickness all over, within which sphere a concentric front moved, nothing would be perceptible. Actually it is more like a hemisphere because the atria are excited earlier and the mass of the atria can in fact be disregarded with respect to that of the ventricles. Furthermore the walls are not of uniform thickness. The left ventricle is about 1.5 cm thick, the right ventricle about 0.5 cm. For this reason we cannot speak of a central symmetrical or even axis-symmetrical spread of the front. In approximation a plane of symmetry can be assumed that halves the left and right ventricles.

The potentials on either side of the plane of symmetry practically cancel each other so that the heart vector moves chiefly in the plane of symmetry.

Finally repolarization necessarily follows depolarization, to prepare the muscle fiber for the next contraction. This repolarization occurs more slowly and moreover not according to the same pattern as the depolarization, but rather the reversed.

## 2. The normal VKG

In consideration of what has been said, it is clear that the form of the VKG cannot be theoretically predicted. The anatomical and physiological data are insufficient, or they are such that even when they are adequate they appear to be too complicated. We must proceed empirically. In the three 140 projections we get approximately the following:

The QRS complex lasts approximately 80 msec. The distribution of the loop in this interval is to be read in the density of the dots (repeat time 2.5 msec).

The QRS complex is preceded by the P apex (auricular contraction) and followed by the T apex (repolarization of the ventricles).

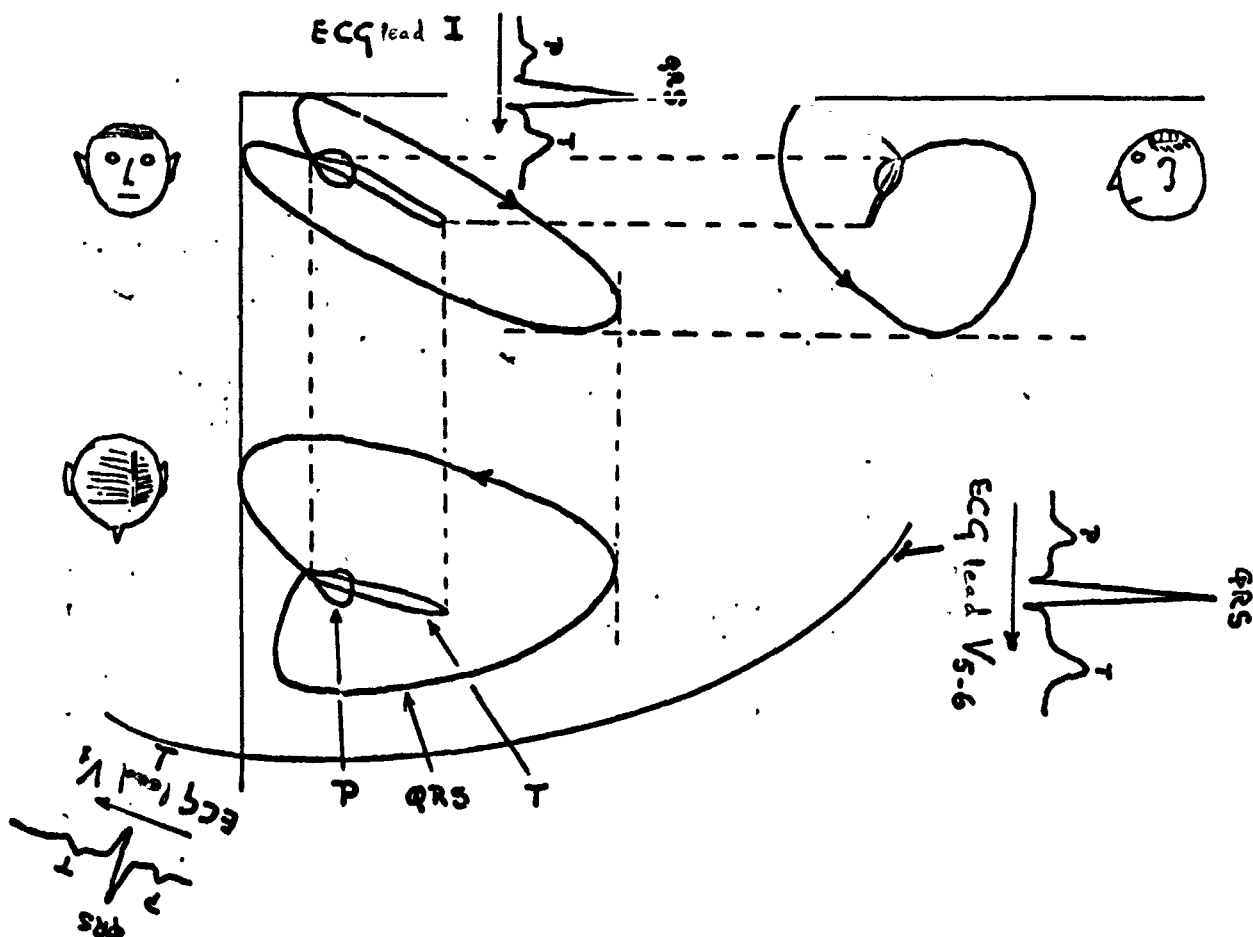


Figure 81. Normal VKG in 3 projections.

The three loops overlap at the origin so that for a good analysis of the central part of the VKG it is necessary to enlarge this portion, or to break off the loop at a suitable point. A normal QRS loop lies mostly in a flat plane. The plane ought to coincide with the plane of symmetry of the heart but actually it deviates from it.

In the frontal projection the QRS vector loop runs "clockwise" in the horizontal and "counterclockwise" in the sagittal. This is clearly an important criterion, typical of the VKG, which is not evident, or only barely so, in the EKG, because in the EKG the temporal relationship between the leads is negligible. The position of the loop surface and the direction of the loop are indicated by the pole vector.

Furthermore in consideration of the projection the "axis" of the loop is important. This is a rather loosely used term in diagnostic evaluation, meaning the preferred direction of the loop. It can be more sharply defined by /141 making use of the direction of the gradient (whose determination from the VKG exacts troublesome measurements and calculations, however) and of the maximum vector.

The relationship between the parts to the left and right of the origin normally  $\sim 90/10$ ) is important also, as well as the relationship between parts that are ahead and behind, above and below.

Finally attention must be directed to special forms, bell curves (in and out), folds, delays, etc., and to the moment of their occurrence.

It is instructive to compare a VKG with a classic EKG (see figure 81). The behavior of the EKG in its different leads is made much more understandable.

### 3. The pathological VKG

Pathological deviations are caused by:

1. Modifications in the mass distribution of the myocardium, either a local "too little" or "too much."

2. Modifications of the time schedule of excitation of the myocardium, in that the moment or the duration of excitation is locally altered.

3. Modification of the conductivity of the environment of the heart or by a combination of these factors.

3.1.1. A "too little" with respect to muscle tissue can arise through infarction. This is based on the blocking of a branch of a coronary vessel. The part of the muscle supplied by this branch dies as a result of the blockage. An "old" infarct (> 6 months, for example) is manifest in the VKG in that there is no deflection in the direction of the diseased part at the moment when it would occur in the case of healthy tissue. In anterior wall infarction for example, the forward-directed vectors in the first 20 msec of the QRS complex are lacking, and with an infarct of the lower wall the initial (30 msec) vectors are directed upward, i.e., away from the diseased part.

In a new infarct there are also specific deflections of the T and S-T segments.

/142

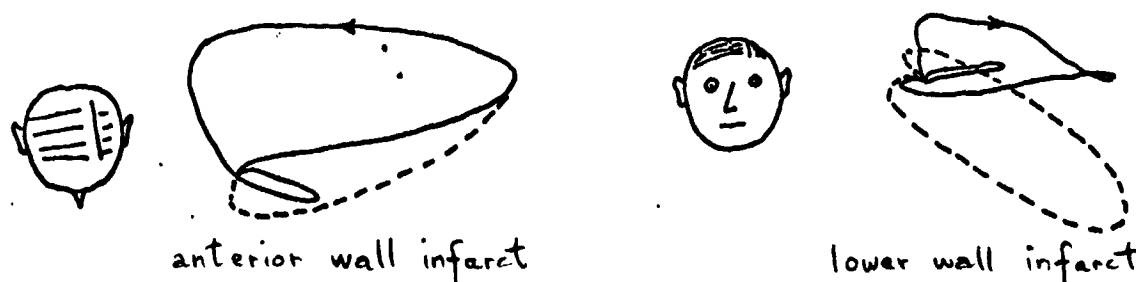


Figure 82. Effect of infarct on the VKG ( is normal).

3.1.2. A "too much" with respect to muscle tissue appears as "hypertrophy" in pressure or volume overload of one of the heart chambers.

In the left ventricle hypertrophy ("left hypertrophy") may be the result, for example, of narrowing of the valve of the aorta (aortic stenosis). In order to maintain a blood pressure that is sufficient for blood supply to the tissue at an acceptable level, the left ventricle must provide a systolic that is much higher than normal, e.g. 240 mm Hg instead of 120 mm Hg. There is thus a pressure overload.

When on the contrary the valve of the aorta is leaky without stenosis (aortic insufficiency) a quantity of blood flows out of the aorta back into the left ventricle during the diastole. This extra quantity of blood must be pumped out at each heart beat without contributing to the circulating volume. This is volume overload, which also causes hypertrophy.

The VKG in left hypertrophy is characterized by the dominance of the vectors at  $\pm 20$  msec which are left-anterior-above. This leads to an increase of the left/right ratio, turning of the QRS "axis" toward the back, counterclockwise movement of the frontal projection and coupled with it a shifting of the pole vector upward and forward. The loop becomes rather pear-shaped.

In serious cases, T and S-T changes may occur which are designated "strain."

The same is true of hypertrophy of the right ventricle. Here pulmonary stenosis or pulmonary insufficiency may be the cause. /143

The VKG of right hypertrophy is characterized by predominance of the vectors at  $\pm 30$  msec which are right-anterior-below. This leads to a decrease of the left/right ratio, turning of the QRS "axis" toward right-anterior, clockwise movement of the horizontal projection and coupled with it an increase of

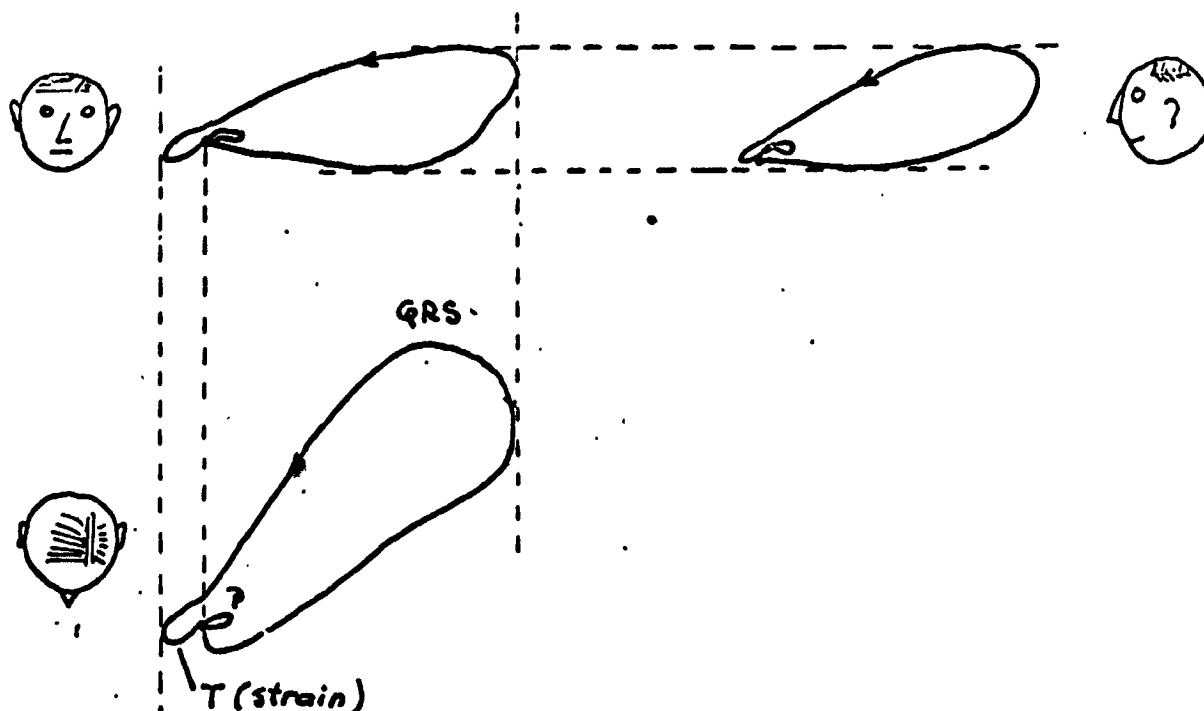


Figure 83. Left ventricle hypertrophy.

the posterior-downward pole vector. The loop becomes somewhat slipper-shaped. In serious cases there are also T and S-T deviations. See figure 84.

Left hypertrophy can be caused by mitral insufficiency and by septum defect as well as by defect of the aortic valve. In both cases the output of the ventricle is diminished (blood is injected into the left auricle or right ventricle) so that volume overload may occur while the circulation level is maintained.

Mitral insufficiency, through damming of the lung circulation, can also lead to right hypertrophy: mitral stenosis has the same effect.

Septum defect also, in addition to left hypertrophy, causes right hypertrophy via volume and pressure overload (left ventricle dams up blood at a high pressure in the right ventricle: ~ 120 mm Hg against ~ 35 mm Hg systolic).

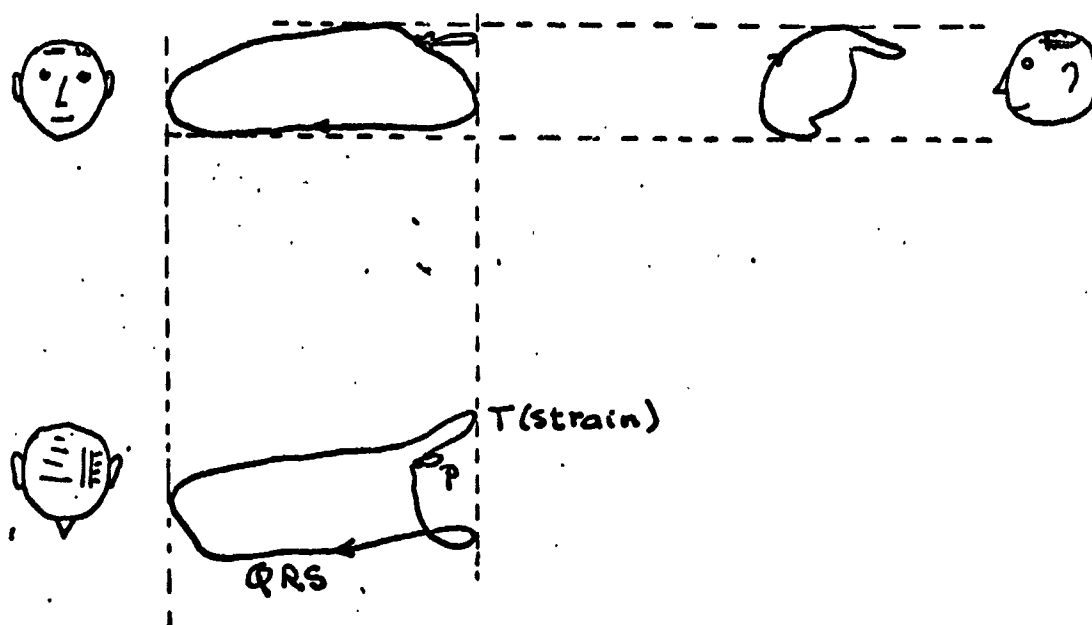


Figure 84. Right ventricle hypertrophy.

An auricular septum defect on the contrary causes right hypertrophy exclusively, by volume overload, because blood flows from the left auricle via the right auricle to the right ventricle which is passively distended more easily than the left ventricle because of its thinner wall. The extra circulating blood volume does not reach the left ventricle and hence causes no overload there.

Hypertrophy of the auricle is also possible (in mitral stenosis or insufficiency, tricuspid valve stenosis or insufficiency, auricular septum defect), but hemodynamically it is of less importance.

3.2. In the case of "too much" muscle tissue, in addition to the increased mass, the increased duration of the excitation in the hypertrophied region also plays a role. Solely by changing the time schedule of excitation, the VKG changes everything else. When this factor is isolated we speak of disturbances of conduction.

When for example the atrioventricular node or the bundle of His does not transmit the stimulus arriving from the atrium, there arises an atrioventricular block. /145

The ventricles can autonomically work against each other (else the patient dies) and this is initiated by a "lower" center of stimulation in one of the ventricles. It should be obvious that the whole pattern of stimulus conduction would then be stood on its head and the VKG changed in a way that is unpredictable.

If the interruption is seated in the right bundle branch (right bundle branch block) then the septum and left ventricle will be excited normally, but the right ventricle would not be excited by the specific stimulus conduction system, but rather by slow muscle conduction. When the left ventricle is excited as a whole, the right ventricle is still depolarizing in an intricate, delayed fashion. The VKG expresses it in a QRS loop of longer duration ( $> 120$  msec for a "complete RBTB") with a characteristic trunklike sluggishly traced deformation toward the right.

By disturbance of the temporal sequence of depolarization, repolarization is likewise disturbed, with a resultant change of the T loop.

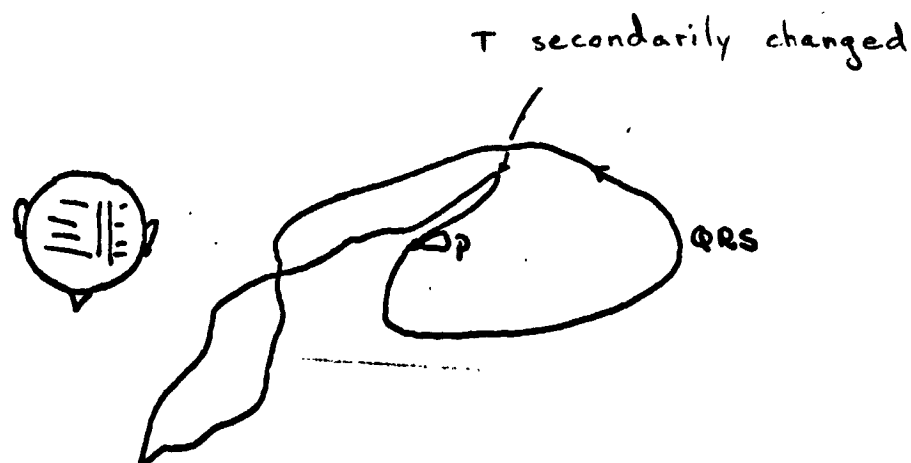


Figure 85. Right bundle branch block.

A left bundle branch block leads by analogous causes to increased duration (> 120 msec: complete "LBTB") and a left-posterior-upward axis. Vectors to the right are almost lacking because excitation occurs from the right bundle branch and the septum which now is excited from the right counterbalances the right ventricle. Secondary T and S-T changes here also. /146

3.3. Changes in the vicinity have the character of increases of electric resistance, for the most part, e.g. in chronic pericarditis (layer of connective tissue around the heart) and in emphysema (lungs retaining air and dilated). The result is "low voltage" in the EKG leads and a small VKG loop.

As a noteworthy example of the application of the VKG in the clinic we mention the auricular septum defect. There are two varieties. The so-called "secundum" (or dorsal) defect and the "primum" (or ventral) defect. The first is uncomplicated, while the second is directed toward the atrioventricular valve and is accompanied by deviations in this valve and sometimes also by a ventricular septum defect. The difference in behavior rests on a different embryonic origin. The primum defect is malignant, more difficult to operate on, and it entails thereby the use of a heart-lung machine. It is thus of great importance that the differential diagnosis between these two be accurate. Here the VKG--quite unexpectedly, moreover--plays an important part.

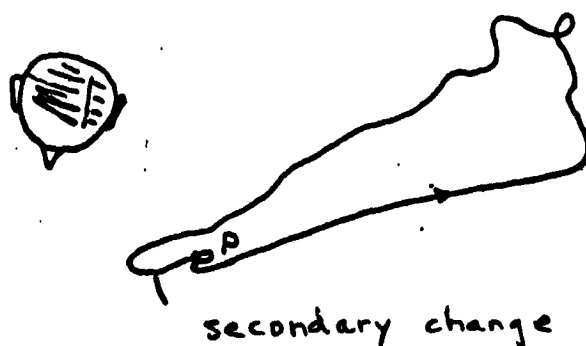


Figure 86. Left bundle branch block.

/147

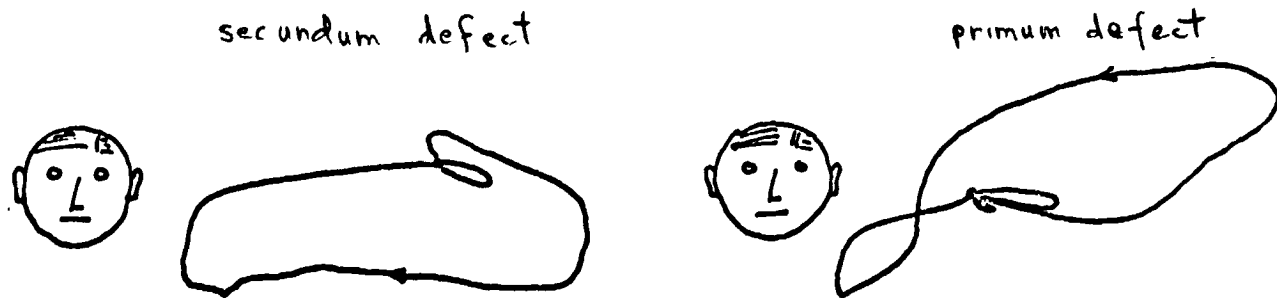


Figure 87. Difference between auricular septum defects.

The two loops are completely different.

In one case the frontal projection moves clockwise, in the other counter-clockwise. The pole vector in the secundum defect is behind the meridian  $90-180^{\circ}$  while the primum defect is ahead. The characteristic aspect of the loops is independent of the presence of hypertrophy and appears to be based on an anomalous structure of the conduction system at the time of embryonic development, together with the developmental anomaly of the septum.

In the EKG this diagnosis can be made with much less certainty because supplying the time relationship between the leads is a troublesome and unreliable operation. It is the intrinsic property of the VKG that it does it well.